Comprehensive Examination
Artificial Intelligence
Spring 2003

1. Consider the following game, which we call Bounded Lying. Player 1 chooses a number between 1 and N (inclusive). Player 2 then guesses the number, and receives one of three possible replies from Player 1: "Yes," meaning that's the number; "greater," meaning "it's > guess; or "less," meaning it's < guess. Two wins if he guesses the number using no more than $\log_2 N$ guesses. If player 1 always tells the truth, then Player 2 can use binary search to win easily. Now suppose Player 1 is allowed to lie k times; that is, he's allowed to say "greater" when the number is less than the guess, or vice versa. If Player 2 names the correct number, Player 1 must admit the truth.

   (a) Is this a game of perfect information?

   (b) Give an algorithm for a computer to play the role of Player Two.

   (c) Give an algorithm for a computer to play the role of Player One.

   By "algorithm" here we do not mean an algorithm that is guaranteed to do as well as possible. Feel free to make use of heuristics.

2. Provide a definition of an edge. Is edge structure dependent on scene structure? Is edge structure dependent on camera resolution? Is edge structure dependent on depth-of-field?

3. Describe two constraint satisfaction problems in computer vision.

4. Consider a simple digital system with two states, called `up` and `down`, and one input, called `button`. Time is discretized, with the first time point of interest called `0`, and the point after point i called `succ(i)`. If the button is not pushed at time i, and the system is in state `up` (resp. `down`) at time i, it will be in state `down` (resp. `up`) at time $i + 1$. If the button is pushed at time i, the system will be in state `down` at time $i + 1$.

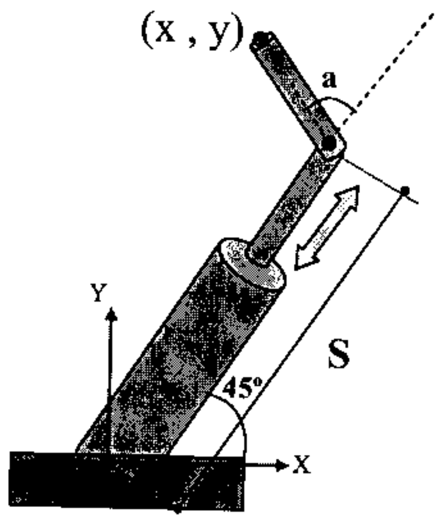
   (a) Produce an axiomatic description of the system. Be sure to describe exactly what symbols you are introducing and exactly what their types are. (That is, for predicates, describe exactly what arguments they take.)

   (b) Using the vocabulary introduced in part (a), formalize the statement: "The button is not pushed before time 3." As we have done here, you can abbreviate `succ(0)` as `1`, `succ(succ(0))` as `2`, and so forth.

   (c) Using any convenient proof technique, prove that if the system is `up` at time `0`, and the button is not pushed before time `3`, then the system will be `up` at time `2`.

   (d) Using the axioms of part (a), can you prove that the system will be either `up` or `down` at time `2`? If not, what further axiom(s) are needed?

5. The robot arm shown below has two degrees of freedom: a prismatic joint and a revolute joint.



The arm is fixed to the table at an angle of 45 degrees from horizontal. The prismatic joint allows the first segment of the arm to extend (increasing the length S). At the end of this first segment is a revolute joint that allows the second segment of the arm to swing through an angle r. The second segment of the arm has a fixed length of 1 meter. The endpoint of the arm is shown in the diagram by the coordinates x,y and the origin of the coordinate frame is shown at the base of the arm.

Part A: The robot is in an initial state (S', r') that results in the endpoint of the arm being at a particular position in space (x',y'). Are there other possible configurations (other values of S and r) that result in the endpoint of the arm being at that exact same position? Why or why not?

Part B: Are there areas of space that the robot cannot reach? Why or why not?

6. Skolemize the following formulas:

   (a) $\forall(x, y)(P(x, y) \Leftrightarrow \exists(w)(P(x, w) \wedge Q(w, y)))$

   (b) $\forall(x, y)(P(x, y) \Rightarrow \exists(w)(P(x, w) \wedge Q(w, y)))$

   (c) $\forall(x, y)(\exists(w)(P(x, w) \wedge Q(w, y)) \Rightarrow P(x, y))$