# Theory Comprehensive Examination
## May 1998

### Question 1 (20 points = 6 + 7 + 7)

(a) Describe a $O(n \log n)$ time algorithm to determine whether two multi sets of $n$ integers each are identical.

(b) Suppose you have found the minimum weighted spanning tree of a connected graph. Now a new edge is added to the graph. What changes do we need to make to the current minimum weight spanning tree to get the MWST for the new problem?

(c) Is the following true: Dijkstra's algorithm for finding single-source shortest paths works correctly provided the graph has no negative length cycle (even if there are negative length edges).

### Question 2 (5 points)

Describe a randomized polynomial time bounded algorithm for the satisfiability problem which has the following properties

(i) if the formula is satisfiable, then the probability that the algorithm returns "satisfiable" is positive, and

(ii) if the formula is not satisfiable, the probability that the algorithm returns "satisfible" is zero.

### Question 3 (20 points = 7 + 7 + 6)

(a) Show either that the following problem is NP-hard or that it is solvable in deterministic polynomial time:

Given $n + 1$ integers $a_1, a_2, \ldots a_n, b$ in binary, is there a subset $S$ of $\{1, 2, \ldots n\}$ with $|S| \leq 10$ so that

$$\sum_{i \in S} a_i = b \, ?$$

(b) At the current state of knowledge, is there a language in NP which is known to be not NP-Complete? Explain briefly.

(c) Suppose for a language $B$, we know that it is in NP and also that $B$ is polynomial time reducible to SAT (the language consisting of satisfiable Boolean formulae). Does this imply that $B$ is NP-complete? Explain briefly.

## Question 4 (20 points = 5 + 15)

Suppose $T$ is a function defined on the nonnegative integers and it satisfies

$$T(0) = 0, \quad T(1) = 1, \quad T(n) = 3T(n-1) - 2T(n-2) \; \forall n \geq 2.$$

(a) Guess what $T(n)$ is (for general $n$).

(b) Prove your answer.

## Question 5 (20 points = 10 +10)

(a) Consider the language:

$$L = \{x : x \text{ is the binary representation of an integer divisible by } 7\}.$$

Argue that this is a regular set.

(b) Is the following language recursive? Explain briefly.

$$\{(M_1, M_2) : M_1, M_2 \text{ are two Turing Machines accepting the same language }\}.$$

## Question 6 (15 points )

Consider the following procedure for finding a matching $M$ in a graph with set of edges $\{e_1, e_2, \ldots e_m\}$: Start with $M = \emptyset$ and for $i = 1, 2, \ldots m$, add $e_i$ to $M$ if it preserves the property that $M$ is a matching. Show that this procedure finds a matching of cardinality at least 1/2 the maximum possible.