Managing a Portfolio of Overlay Paths

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ABSTRACT

In recent years, several architectures have been proposed and developed for supporting streaming applications that take advantage of multiple paths through the network simultaneously. We consider the problem of computing a set of paths and the relative amounts of data conveyed through them in order to provide the desired level of performance for data streams. Given the expectation, variance, and covariance of an appropriate metric of interest for overlay links, we attempt to solve the underlying resource allocation problem by applying methods used in managing a finance portfolio. We observe that the flow allocation problem requires constrained application of these algorithms, and we discuss the tractability of enforcing the constraints. We finally present some simulation results to evaluate the effectiveness of our proposed techniques.

Categories and Subject Descriptors
C.2.2 [Computer Communication Networks]: Network Protocols—Applications, routing protocols

General Terms
Algorithms, Performance, Measurement, Experimentation

Keywords
Overlay networks, video streaming

1. INTRODUCTION

In recent years, considerable work has been done in characterizing the quality of redundant paths in the Internet. Recent work on overlay networks has allowed various projects to exploit this redundancy to achieve better performance and higher fault-tolerance. Detour [24], for instance, improves routing efficiency by exchanging congestion information between nodes and adaptively routing through overlay paths that correspond to better routes. Resilient Overlay Network (RON) [1] allows distributed applications to perform overlay path selection in an application-specific manner, detect path failures, and recover by routing data through other overlay paths. In fact, many high performance streaming media systems exploit the redundant paths in a concurrent manner to provide higher throughput or better quality of service. CoopNet [20] streams media and employs striping over multiple overlay trees to enhance both performance and reliability. Splitstream [7], which is built on top of the Pastry overlay network, also multicasts content over multiple overlay trees. Byers et al. [6] propose a system for performing multi-point transfers across richly connected overlays by judiciously coordinating delivery of subsets of data in a highly distributed and concurrent fashion. Cheng et al. [8] address the dual problem of collecting data from several hosts by carefully scheduling the movement of data. Nguyen and Zakhor [19] propose the use of sending redundant data over multiple paths to minimize packet loss.

An important issue that repeatedly shows up in many of the above-mentioned efforts is the need to identify a good set of redundant paths to be used for communication. In order to obtain good performance, most systems try to use overlay paths that correspond to disjoint sets of physical links. In situations where completely disjoint paths can not be identified, these system try to minimize the use of shared physical links [19, 31]. Typically, a probing tool, such as traceroute, is used to obtain the characteristics of the overlay paths, and some simple heuristics are employed to estimate the disjointness of candidate paths during path selection. Nguyen and Zakhor [19] use a metric that counts the number of shared physical links between two candidate paths, while Zhang et al. [31] take into account the latency of the shared physical links in order to distinguish low-latency LAN links from high-latency backbone links. Given their simplicity and their heuristic nature, these schemes could fail to recognize that some of the physical links are actually high-performance, over-provisioned links and would refrain from using paths that share them. Splitstream makes an indirect attempt at minimizing the sharing of physical links by ensuring that each overlay node appears as an intermediate node at most once in different overlay trees. Many other user-level multicast proposals [4, 14, 22, 33] recognize the importance of this issue and evaluate their systems us-
ing performance metrics that track the worst case number of overlay paths that share some physical link in the network. They do not, however, provide any specific techniques for minimizing this quantity.

In this paper, we propose methods for computing a set of paths and the relative amounts of data to be conveyed through them for a given pair of source-destination nodes. We require as inputs statistical measures that characterize the behavior of the overlay links with respect to a predetermined performance metric, such as latency, jitter, or loss-rate. The statistical measures used in our approach are traditional ones such as expectations, variances, and covariances. For instance, in one of our target settings, where we are interested in optimizing for user-perceived latency, we take into account the mean and variance of latency for each overlay link as well as the covariance of latency over pairs of links. We believe that the complex physical structure of the underlying network could be captured by these statistical measures. In this respect, we draw inspiration from previous work [3, 12, 23] on shared congestion detection that is based on the observation that if two flows share a congested physical link, then packets from the two flows traversing the congested link at about the same time are likely to be either dropped or delayed by similar amounts of time. The correlation statistics could be computed by either actively injecting probing packets or passively monitoring the behavior of real content sent over multiple paths. We then solve the resource allocation problem of computing what overlay paths are used and what fractions of data is sent over each path in order to satisfy a desired level of performance and a desired tolerance to variability in performance. The techniques used to solve the resource allocation problem are inspired by methods used in portfolio management that attempt to perform allocation across assets when the assets have complex interactions and correlations.

2. APPLICATION OF FINANCE MODELS TO OVERLAY NETWORKS

In this section we consider the applicability of theories from finance domain to overlay networks. We focus on Portfolio Theory that is based on the mean-variance model and solves the problem of optimal portfolio selection for an individual. We do not consider the Capital Asset Pricing Model (CAPM), which builds on the mean-variance analysis to characterize asset prices in market equilibrium, since it relies on a rather restrictive set of assumptions [16, 25]. We look at the basics of Portfolio Theory and determine how to translate it to the domain of overlay networks.

Portfolio Theory [2, 30] was introduced in 1950’s by Harry Markowitz and further developed by Merton Miller and William Sharpe. The theory explores how risk-averse participants construct portfolios in order to optimize expected returns for a given level of market risk. The theory relies on the fact that participants’ utilities are completely characterized by mean and variance of expected returns. An efficient frontier can then be constructed and each portfolio on the frontier will offer the maximum possible expected return for a given level of risk.

As an example, consider a candidate set consisting of just two securities, Exxon and Southwest Airlines, with one of the stocks (Southwest Airlines) exhibiting higher expected returns and higher variance in returns (as graphed in Figure 1). A cautious investor who prefers lower returns over higher risks would choose an allocation that consists of a greater number of Exxon shares, while others, who are willing to take more risks, will choose a portfolio consisting of a larger number of Southwest Airlines shares. Furthermore, if the returns from the two stocks are not perfectly correlated, then one could design a portfolio that exhibits a lower level of risk than either one of the individual stocks.

In general, given the mean, variance, and covariances of the returns from stocks, one can compute the maximum possible expected return for a given level of risk as well as construct a portfolio that minimizes the risk for a desired expected return. Figure 2 graphs the expected return and its standard deviation for different combinations of two stocks with different levels of correlations. The two stocks in isolation have expected returns of 10% and 20% and standard deviations of returns of 15% and 25%. When the stocks are perfectly correlated (as in the rightmost curve), a portfolio of two stocks exhibits a strictly linear behavior with respect to expected returns and standard deviation of returns. When the stocks exhibit perfect inverse correlations (as in the lefmost curve), it is possible to design a portfolio that exhibits an intermediate value for the expected return with very little or even zero risk.

The mean-variance model discussed above could be adapted to perform path selection in overlay networks. Given paths with different levels of quality, one could use some combination of the paths to minimize the fluctuations in quality. A selection process (based on the mean-variance model) would invariably use uncorrelated paths to minimize the variability in performance. Paths that do not share low-performance physical links are precisely those that exhibit low correlations in performance and would get picked by the selection process that minimizes variance. Such paths are unlikely to suffer from simultaneous drops or packet delays making

Figure 1: Return profiles for Exxon and Southwest Airlines.
them suitable for various multi-path schemes that transmit redundant data, such as the coding scheme used by Nguyen and Zakhor [19] or the digital fountain scheme used by Byers et al. [6]. Furthermore, our methods not only compute the paths to be used in a multi-path transmission, but also provide guidance on what amounts of data are to be sent on different paths.

Portfolio theory, however, makes a number of assumptions, which we will need to translate from the domain of finance to overlay networks. In particular, the following assumptions deserve some analysis.

**Mean-Variance:** Participants base decisions solely on expected return and variance. For this assumption to hold, we need to verify that user satisfaction depends completely on the expected mean and variance of metrics that characterize user experience with streaming multimedia. The literature that examines user experience with streaming audio and video shows that metrics such as latency (delay), jitter, and loss-rate [9, 10] have the most significant impact on perceptual quality of streaming multimedia. In particular users are more satisfied when mean latency, jitter (which is the variation in packet delay) and packet loss rate are minimal. Thus we can conclude that users care the most about the expected mean and variance of latency, where delay in transmission and packet loss rate contribute to average latency of the link and jitter can be seen as a variability in the delay for packets.

**Risk Averse:** Participants require higher returns for higher risk and are willing to tolerate lower returns to achieve lower risk. This assumption is again plausible. Studies in user perception of video and audio streams showed that even though users prefer higher multimedia quality, they choose lower bit rate over higher packet loss rate if the latter makes the visual and audio information too distorted to comprehend [28]. Thus users are willing to tolerate lower throughput in exchange for lower jitter. However, if there is a possibility to avoid or diminish negative effects of jitter (for example by buffering multimedia streams), users will tolerate higher packet loss rate in exchange for higher throughput.

**Rational:** Participants maximize one-period expected utility, and utility is solely a function of mean and variance. This assumption holds if the appropriate metrics for overlay links/paths are considered when defining users' utility functions. As mentioned before user experience with data streams depends on metrics such as throughput and jitter. If these are the only concerns of users then mean and variance of these metrics can be seen as the most important characteristics of a link/path. Thus users can maximize their utility functions by choosing paths that minimize the above mentioned metrics. If another important metric is of concern (for example monetary cost of the link/path, reliability, security, etc), this metric can be incorporated into users' utility functions (see [29] for an example). Then an optimization of a combination of metrics can be performed in order to maximize utility.

**Predictive Constancy:** Statistical measures are valid for some reasonable period of time. A participant is assumed to make exactly one portfolio decision, which remains unchanged for a given time period. The assumption holds if during this time period the estimated statistical measurements of the properties of links/paths are close to actual. Previous networking studies, such as the one by Zhang et al. [32], have shown that the Internet paths display a surprising amount of predictive constancy. Moreover, if the network is fairly dynamic, statistical probes can be performed more frequently and information gathered from probes can be gainfully employed for future resource allocation decisions.

**Individual Behavior in a Large System:** The decisions of any one participant do not affect asset prices. We model the network as consisting of a large number of nodes operating independently. This is an assumption that is well-justified in the case of the Internet both at an IP level as well as AS level. Further we assume that the network is in a regime that is sufficiently under capacity such that individual flows cannot affect the long run distributions of network parameters such as latency and throughput. With regard to the Internet this additional assumption though not universally true, can be motivated on the following grounds. ISPs shape traffic so as to contain flows within specific limits; for example by they can clip flows that surge over a particular threshold. Many applications also self-shape; for example rich-media applications, such as Windows Media, adaptively thin the stream to guarantee a specified quality of service. Furthermore, user behavior self-selects to be in a stable operating regime; it has been confirmed by multiple studies that users abandon congested sites and services [15].

### 3. FORMULATION

Our formulations reflect the fact that real-time multimedia applications require low latency and/or low loss-rate and that variations in throughput or latency serve to detract from the end user's experience. Given a collection of overlay paths it is natural to model the throughput and latency of each path as a random variable with a given distribution. Then the problem of optimizing the end-user's experience reduces to that of trying to find the optimal allocation of packets to the given collection of paths so as to minimize variation, subject to a given choice of average latency or collective throughput. Below we formalize different variants of these problems and present corresponding solutions. We attempt to construct statistical measures for overlay paths by composing the statistical measures of the individual overlay links that comprise an overlay path. (We refer to the physical path between two end-systems as an overlay link, and a sequence of overlay links as an overlay path.) Once we compute a solution that determines a set of overlay paths with associated weights referring to fractions of the data flow to be conveyed through a certain path, we then assume that the transport process would transmit packets in a ran-

**Figure 2:** Mean-variance frontiers for two securities with streaming audio and video shows that metrics such as provide guidance on what amounts of data are to be sent on different paths.

**Expected Return**

**Standard Deviation**

- 5%
- 10%
- 15%
- 20%
- 25%
- 30%
- 50%
- 75%
- 100%
dom and independent manner down the various paths with a probability that is proportional to their weights.

3.1 Latency

In this section we look at two formulations that take latency into consideration. At first we look at latency as a random variable from a general distribution. We are interested in the first two moments of this distribution - the mean and variance. (The rationale for these two metric being of particular importance is discussed in the previous section.) Later we make more restrictive assumptions about the distribution of latency.

3.1.1 Latency without cut-off

Consider a collection of links \( i = 1 \ldots n \) with latencies \( L \). Let the covariance matrix be represented by \( C; C_{ij} = \text{covar}(L_i, L_j) \). Let \( w \) represent the vector of weights, where \( w_i \) is the fraction of data sent along link \( i \). Let \( \mu \) be the desired average latency. Here we assume that latency is a fungible commodity; in other words packets that arrive earlier than the average compensate for late packets.

Then we wish to solve the following optimization problem (where \( w' \) stands for the transposed version of \( w \)):

\[
\text{Optimization Problem 3.1}
\]

\[
\begin{align*}
\text{minimize} & \quad w' C w \\
\text{subject to:} & \quad w' L = \mu \\
& \quad w' 1 = 1 \\
& \quad w \geq 0
\end{align*}
\]

This problem can be solved to desired precision in polynomial time [26], since quadratic programming with a positive semi-definite matrix is in \( \mathcal{P} \). (A symmetric matrix \( M \) is said to be positive semi-definite if all its eigenvalues are nonnegative, or equivalently for all vectors \( x \) it is the case that \( x' M x \geq 0 \). It is easy to see that any covariance matrix \( C \) is positive semidefinite since \( x' C x \) is just the variance of the collection of links weighted by \( x \) and hence nonnegative.)

3.1.2 Latency with cut-off

Another scenario is where we have a cut-off \( T \) and packets that arrive later than the cut-off are discarded. A natural measure to minimize is the percentage of packets that are discarded. In this formulation we model latency of a link as a normally distributed random variable. This is a reasonable assumption to make considering that link delays can result from a variety of diverse independent phenomena ranging from the physical to queuing. We make use of the fact that linear combinations of normal variables are also normal. This allows us to compute the latency of a path as the sum of the latencies of the individual links. It also allows us to compute the covariance between paths in terms of the covariance between the corresponding links.

Consider a weighted combination of the links, as in the previous subsection, with average latency \( \mu \) and standard deviation \( \sigma \). Then since this combination is normally distributed we know that the percentage of packets discarded is proportional to \( \exp - \left( \frac{\overline{L} - \mu}{\sigma} \right)^2 \). This percentage is visually represented in Figure 4 as the tail of normal distribution to the right of the vertical cut-off line.

But this means that for a given cut-off \( T \) we wish to find the portfolio that minimizes \( \frac{n T - \mu}{\sigma} \). Consider the set of graphs in Figure 3 of \( \sigma \) versus \( \mu \). In the case where we do not have the constraining inequality \( w \geq 0 \) (left portion of Figure 3), we know from finance that \( \sigma \) is a quadratic function of \( \mu \). Figure 3 (middle) represents the corresponding situation in networks domain. It is clear from this figure that the point that minimizes the percentage of discarded packets is the point on the curve where the tangent with negative slope passing through \((0, T)\) touches it. This is a remarkably simple characterization that is analogous to an equivalent situation in finance. The equivalent of the cut-off in networks is the riskless asset in finance, which defines the market portfolio as the point of tangency with the analogous tangent of positive slope.

The optimum collection of weights is easily calculated in the case where we do not have the constraining inequality since the equation of the quadratic can be exactly calculated using the method of Lagrange multipliers. In the case where the weights are restricted to be non-negative, the optimum can be easily calculated as well. In this situation we get only a section of the quadratic and have to consider two cases. In the first case the optimum lies in the interior of the quadratic (middle Figure 3), and then is the unique global optimum that can be easily calculated as in the case with no constraining inequality. In the second case the optimum lies at one of the two endpoints (right Figure 3), which can also be easily checked. Once we determine the optimum it is easy to calculate the collection of weights for the paths to achieve this optimum. Thus this problem is soluble in \( \mathcal{P} \).

3.1.3 Jitter

Jitter is defined to be the inter-arrival delay between successive packets. It is easy to see that the average jitter is 0, since our model is that packets are drawn independently. Of course this assumes that the delay between successive packets at the transmitting end is negligible. Now consider the variance of jitter, since jitter is the difference between two draws (representing latencies) from the same distribution, if \( \mu \) and \( \sigma \) are the mean and standard deviation of the distribution, then the variance of jitter is twice the variance of the resulting latency. Hence the problem of finding the minimum standard deviation of jitter can be represented as Optimization Problem 3.1 and can be exactly solved in \( \mathcal{P} \).

3.2 Throughput

Based on statistical measurements and studies of Internet traffic [11,17,27], we consider throughput to be log-normal (for description of lognormal distribution, please see [13]
Consider again Optimization Problem 3.1 but now let \( \mathbf{L} \) represent the vector of throughputs of the paths. As before we can solve this minimization problem in \( \mathcal{P} \). An interesting point to note in the above formulation is that the vector of means need not be for the same metric as the matrix of covariances. In other words, we could also solve the problem of minimizing the variance of throughput subject to a specified average latency. In general, we can solve any optimizing function that is the linear combination of the first and second moments of two different metrics.

4. EXPERIMENTS

We conduct trace-driven simulations to study the effectiveness of our proposed techniques for communicating video streams. Here we present the results for our latency formulation. We provide preliminary anecdotal evidence regarding the utility of our methods. We quantify how much improvement the system is able to provide over a single-description (SD) and multiple-descriptions (MD) encoded streams with a standard MPEG-2 encoder at 140Kbyte/s.

We use time-domain partitioning method with two descriptions and a simple concealment technique that uses the corresponding frame in neighboring frames (the other description in the MD case) or just repeats the information from the last available frame. More details regarding how the multiple descriptions are generated could be found in [5, 18].

We compare the following path selection algorithms. We first select the shortest path (based on probed data) and consider the transmission of a single description over this path. We refer to this scheme as “Single Shortest Path.” For “Link Disjoint Path,” we generate all paths (of bounded overlay hop count) between two nodes, consider pairs of paths that minimize sharing of links, and choose a pair with the lowest latency. For “Optimal Portfolio Paths,” we use the statistical information gathered from probes and solve the problem formulated in Section 3.1.1. In addition, we also consider a scheme (referred to as “Equal Usage Paths”) where we solve the problem formulated in Section 3.1.1 with the additional constraint of requiring the selection of just two paths with predetermined fixed weights of 0.5 each. Figure 3.2 graphs the PSNR values for different frames transmitted during a single session for different path selection algorithms. (The average over the entire sequence is displayed at the top right corner of each graph.) The paths that are obtained from solving the variance optimization problems perform better than the simpler link-disjoint paths, especially when the system exhibits bursty losses.

5. CONCLUSION AND FUTURE WORK

In this paper, we consider the applicability of optimization techniques used in Portfolio Theory to the problem of path selection in overlay networks. We consider different formulations that attempt to optimize for different performance metrics, with each formulation attempting to attain a desired level of performance while minimizing the variance of either the same metric or that of a different metric. This paper represents just the first step in our overall goal of exploiting path diversity. Future work includes an in-depth evaluation of possible gains and will be performed in both the trace-driven simulator as well as realistic overlay networks, such as the Planetlab.

6. REFERENCES

Figure 5: Simulation result of FOREMAN sequence