Relationships Between Broadcast and Shared Memory in Reliable Anonymous Distributed Systems

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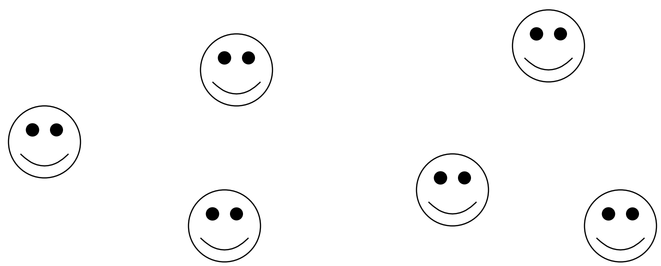
Anonymity

Processes do not have identifiers and execute identical programmes.



Advantages

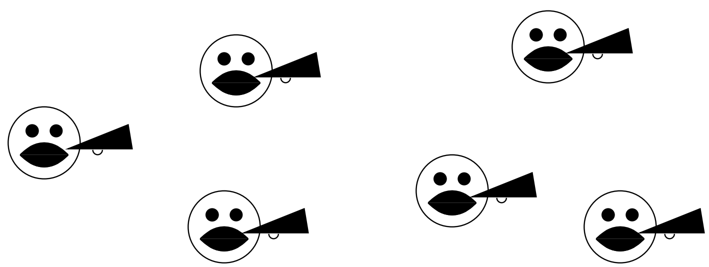
- cheaper to mass-produce
- less testing required
- can enhance privacy

Disadvantage

Problems that require symmetry-breaking become impossible.

Anonymous Broadcast

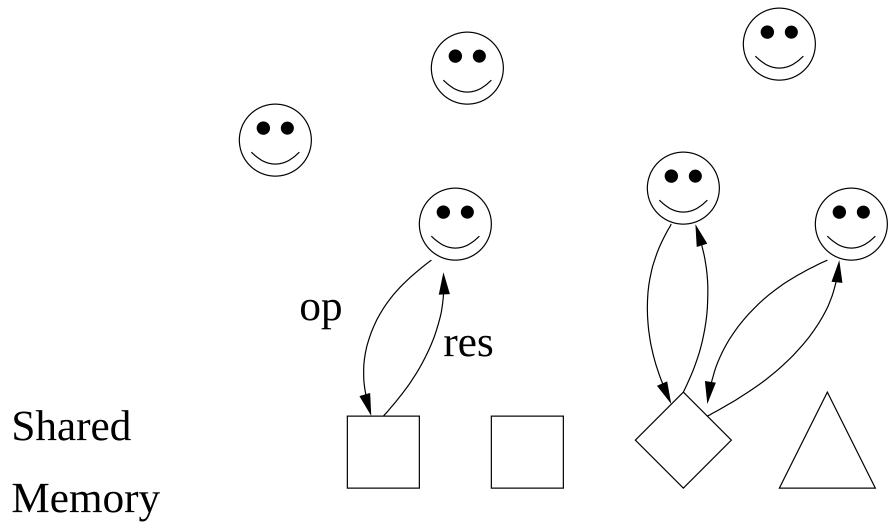
The processes communicate by broadcasting messages to all processes.



No failures, n known to all processes.

Anonymous Shared Memory

Processes communicate via shared objects of various types.



- No failures, n known to all processes.
- Linearizable objects, initialized as programmer wishes.
- Provides abstractions that are useful for programmer.

Question: How do shared-memory models relate to broadcast model (and one another) in anonymous systems?

Previous Work on Anonymity

• Some research on point-to-point message-passing systems.
Impossibility of leader election in a ring [Angluin, 1980].
Using asymmetry of network to solve problems [Boldi, Vigna, 1999–2001].

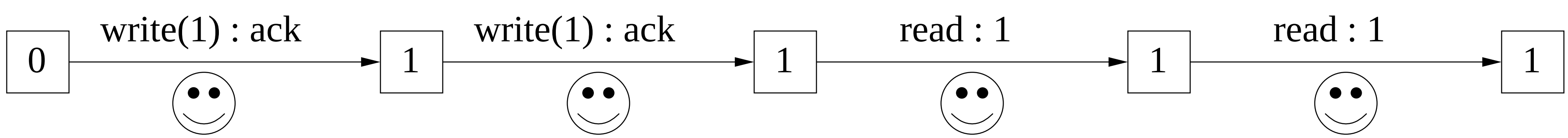
• Agreement tasks solvable using registers (no failures) [Attiya, Gorbach, Moran, 2002].

• Naming possible, but not consensus, using randomization and registers (halting failures) [Buhrman *et al.* 2000].

• Topological approach used to characterize tasks with wait-free solutions from registers [Herlihy, Shavit 1999].

Idemdicence

A shared object is idemdicent ("same-saying") if two consecutive invocations of the same operation (with the same arguments) always return identical responses.



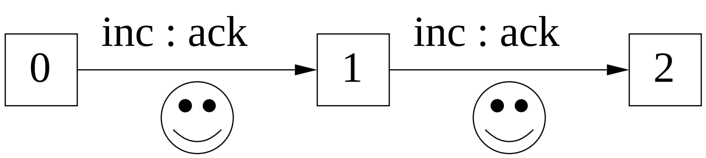
Idemdicent objects: registers, snapshots, consensus objects.

Non-idemdicent objects: queues, compare&swap.

More Examples

Counter

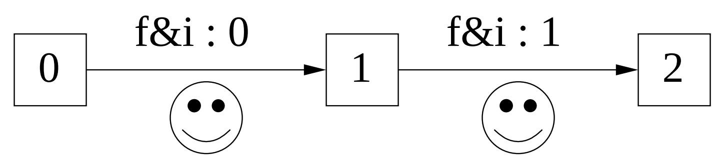
- stores an integer
- increment operation
  (returns ack)
- read operation



Counters are idemdicent.

Fetch&Increment

- stores an integer
- single fetch&inc operation
  (returns value)



Fetch&Inc objects are not.

What Can Broadcasts Implement?

Theorem Broadcasts can simulate shared objects iff the objects are idemdicent.

($\Rightarrow$): We show that asynchronous broadcasts can simulate a synchronous system that contains any idemdicent objects.

Store a local copy of shared memory at each process.

To simulate round r:
A process that wants to perform op on object X broadcasts (r, op, X).
Each process collects all n broadcasts for round r
and simulates all operations on the shared objects locally.

If several processes access same object in the round, order operations lexicographically.
Idemdicence $\Rightarrow$ no need to break ties consistently.

The Converse

Theorem Broadcasts can simulate shared objects iff the objects are idemdicent.

($\Leftarrow$): We show that even synchronous broadcast cannot implement an asynchronous system with a non-idemdicent object.

Consider a non-idemdicent object.
If all processes perform same operation on it, at least two will get different results.
Synchronous broadcasts cannot break symmetry in this way.

Broadcast $\equiv$ Counters

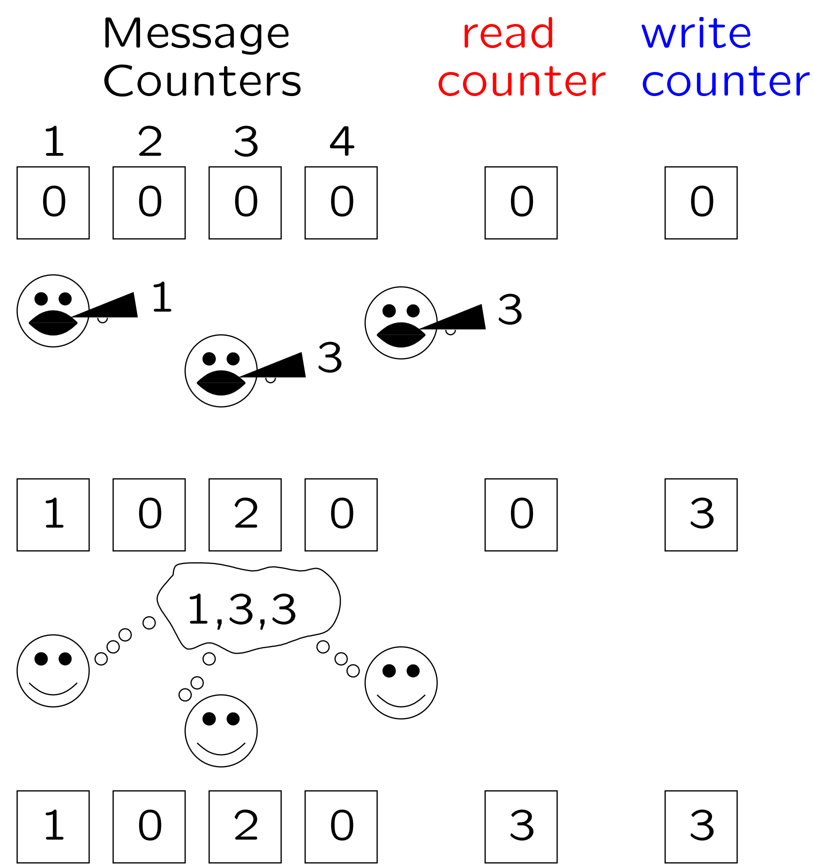
Counters are idemdicent, so broadcasts can simulate them.

Conversely, we show how the asynchronous counter model can simulate synchronous broadcasts.

WLOG, assume bounded-length messages.

Use one counter for every possible message that can be sent plus a read counter and a write counter.

Counters Simulate Broadcasts



To send a message, increment the corresponding message counter and then the write counter.

Wait until write counter mod $n = 0$.
Read all message counters.
Increment read counter.

Wait until read counter mod $n = 0$.
Start next round.

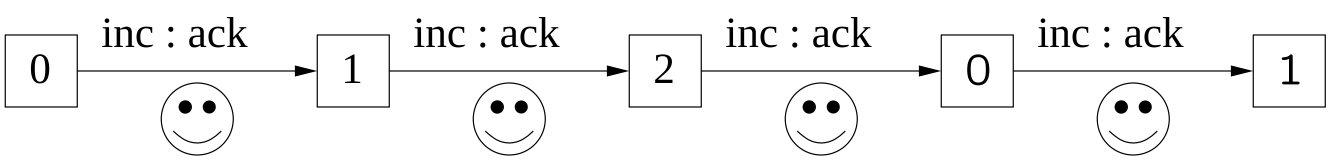
Idempotence

An object is m-idempotent if

- it is idemdicent, and
- doing an operation $m+1$ times has same effect as doing it once.

Examples A register is 1-idempotent.

A mod-3 counter is 3-idempotent:



Clones

A collection of processes behave as clones in an execution if

- they have the same input,
- they run in lock-step, and
- all perform the same step in each round.

If objects accessed by P in an execution are m-idempotent, we can add m clones of P to the execution, and nobody will notice their presence.

Configurations C', C'' are indistinguishable (except to the two clones) if objects accessed are 2-idempotent.

Broadcast is Stronger than Registers

Registers are idemdicent, so broadcasts can implement them.

Threshhold-2 function:

- binary function of n variables
- output is 1 iff at least 2 inputs are 1.

Easy to compute using broadcast.

Impossible to compute (even synchronously) using registers (when $n > 2$) since no register-based algorithm can distinguish

2 clones with input 0,
1 process with input 1

from

1 process with input 0,
2 clones with input 1

Robustness

Robustness is a desirable property of shared-memory models.

It says objects that are weak when used individually are no stronger when used together.

I.e. types A and B can implement type C only if
A alone can implement C or
B alone can implement C.

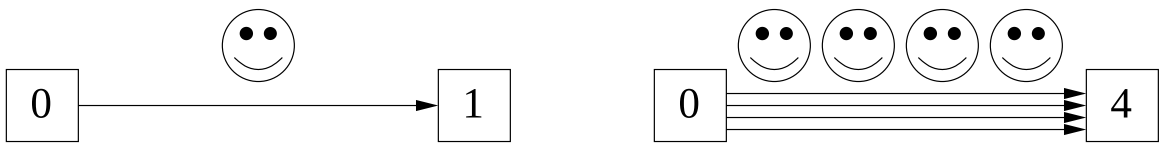
Lots of research on robustness in asynchronous, wait-free models.
Robustness violated by somewhat strange objects.

Here we have a natural counter example to robustness:
mod-2 counters and mod-3 counters can be used together to count up to 5.

mod-3 Counters Cannot Count up to 5

Consider any algorithm constructed using mod-3 counters.

Since mod-3 counters are 3-idempotent,
4 clones are indistinguishable from a single process.



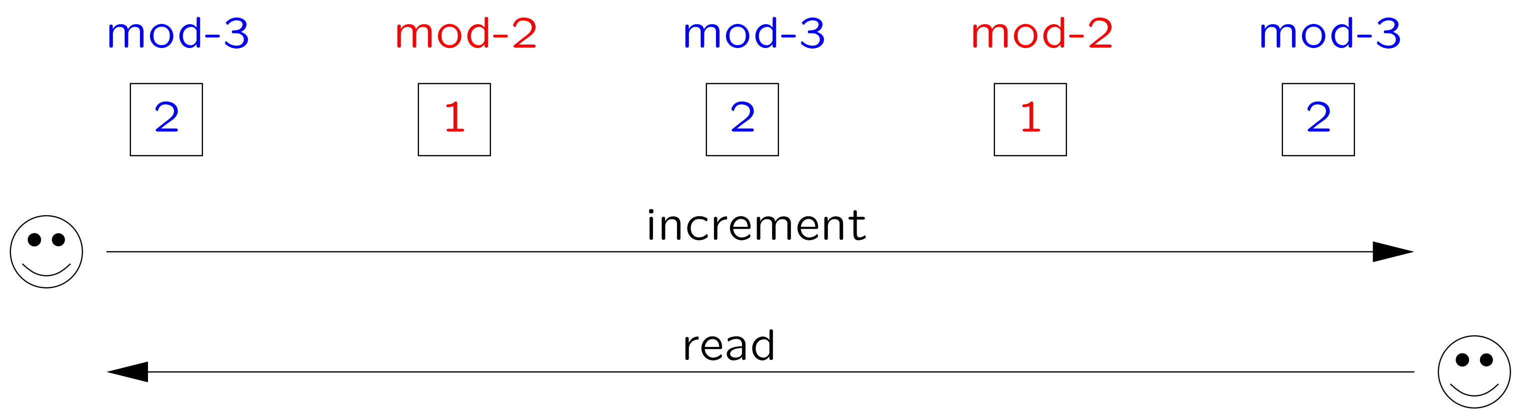
Cannot tell if the value of an up-to-5 counter should be 1 or 4.

Similarly, mod-2 counters cannot count up to 5.

mod-3 + mod-2 Counters Can Count to 5

Our definition of an "up-to-5 counter" says correct responses required only when fewer than 6 increments occur.

Use 3 mod-3 counters and 2 mod-2 counters arranged in a row.

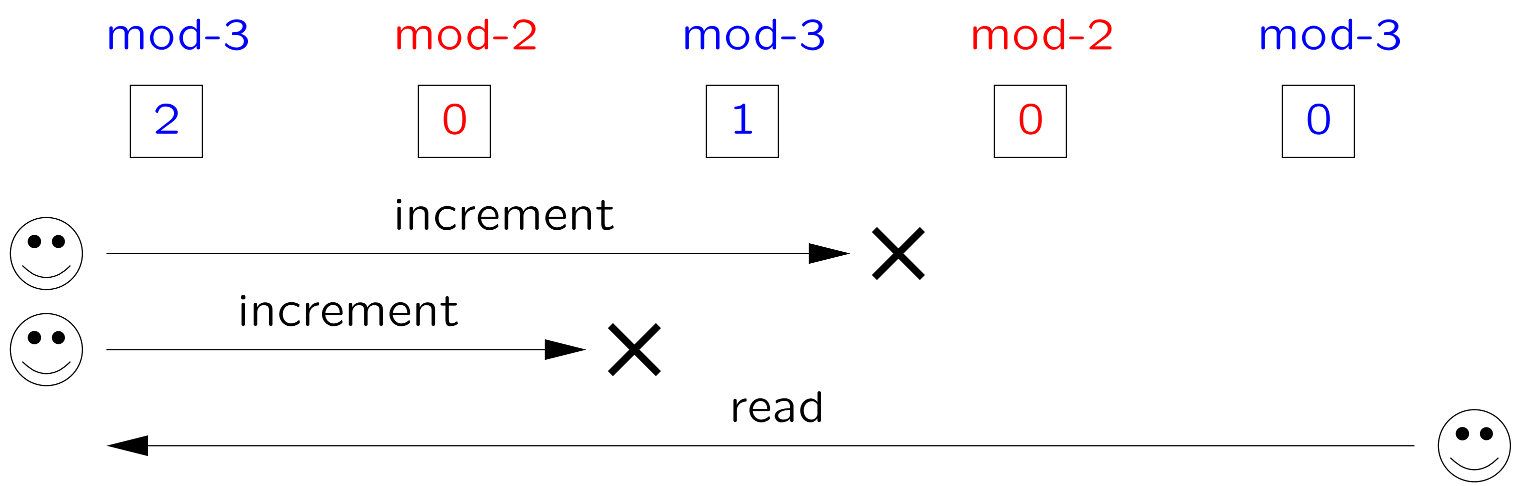


A set of reads is consistent if all mod-3 counters return equal responses and both mod-2 counters return equal responses.

Read repeatedly until you get a consistent set.
Return unique value in $\{0, 1, .., 5\}$ that could give these values.

Termination

Inconsistent sets of reads are caused when a read "crosses" an increment.



No process will do more than 5 increments.
Eventually all increments will terminate, and consistent set will be obtained.

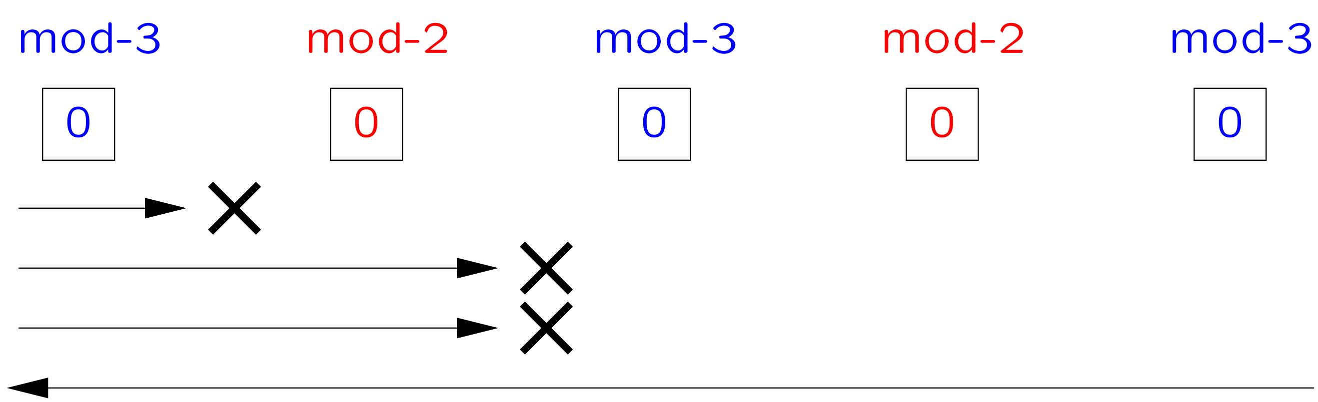
Correctness

Prove correctness only when fewer than 6 increments are done.
Linearize all operations when they (last) access middle object.

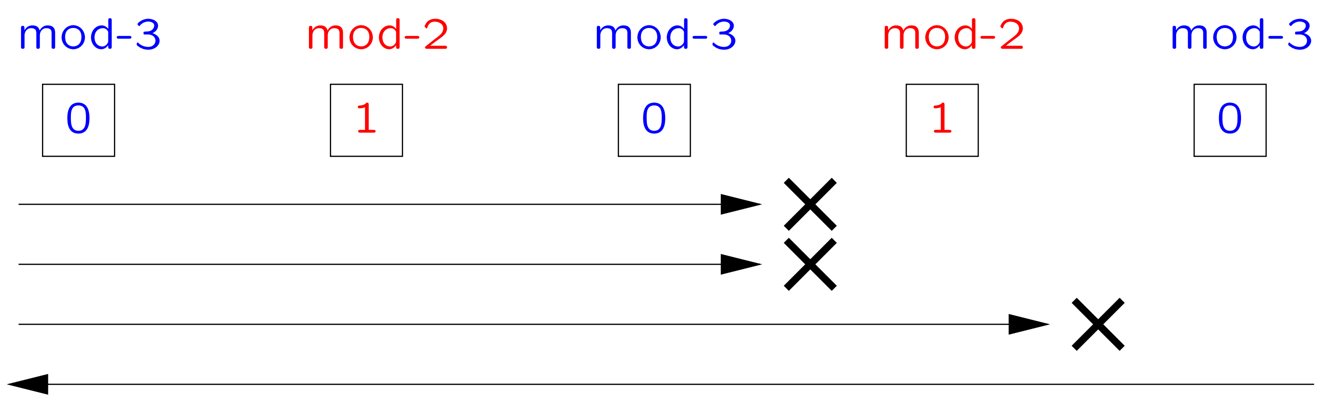
If a set of reads is consistent, there are three cases.

Case 1: No increments in progress.

Case 2: 3 increments are crossed in left half.



Case 3: 3 increments are crossed in right half.



In each case, the values that were read tell us exactly how many increments accessed the middle counter.
$\Rightarrow$ Linearization is correct.

Generalizing the Construction

Theorem Let $m = \text{lcm}(m_1, \ldots, m_r)$.
There is an implementation of an m-valued counter from the set $\{\text{mod} - m_1 \text{ counter}, \ldots, \text{mod} - m_1 \text{ counter}\}$.

Proof uses a larger array of various counters, and the Generalized Chinese Remainder Theorem.

Open Questions

What is computable when n is unknown to processes?

What about models with failures?

How can anonymity be used in a practical way to help protect privacy?