Problem Set 1 Solutions

1 Problem 1.1.28

For parts (a) and (b) we have

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$(p \vee q) \wedge r$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	T	F
F	F	F	F	F	F

For parts (c) and (d), we have

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$	$(p \wedge q) \wedge r$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	F	T	F
F	F	F	F	F	F

For parts (e) and (f), we have

p	q	r	$\neg r$	$p \wedge q$	$(p \wedge q) \vee \neg r$	$p \wedge q$	$(p \wedge q) \vee \neg r$
T	T	T	F	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
T	F	F	T	T	T	F	T
F	T	T	F	T	F	F	F
F	T	F	T	T	T	F	T
F	F	T	F	F	F	F	F
F	F	F	T	F	F	F	T

2 Problem 1.2.20

$$\begin{aligned}(p \to q) \wedge (p \to r) &\equiv (\neg p \vee q) \wedge (\neg p \vee r)\\ &\equiv \neg p \vee (q \wedge r)\\ &\equiv p \to (q \wedge r).\end{aligned}$$

The first equivalence follows from the logical equivalence involving implications, the second equivalence follows from the distributive law, and the third from the logical equivalence involving implications.

3 Problem 1.3.58

a. $\forall x(P(x) \to \neg S(x))$.

b. $\forall x(R(x) \to S(x))$.

c. $\forall x(Q(x) \to P(x)$.

d. $\forall x(Q(x) \to \neg R(x))$.

e. Yes. If x is one of my poultry, then he is a duck (by part (c)), hence not willing to waltz (part (a)). Since officers are always willing to walts (part(b)), x is not an officer.

4 Problem 1.4.26

a. This is false, since $1+1 \neq 1-1$.

b. This is true, since 2+0=2-0.

c. This is false, since there are many values of y for which $1+y \neq 1-y$.

d. This is false, since the equation $x+2=x-2$ has no solution.

e. This is true, since we can take $x=y=0$.

f. This is true, since we can take $y=0$ for each x.

g. This is true, since we can take $y=0$.

h. This is false, since part (d) was false.

i. This is certainly false.

5 Problem 1.5.12

a This is correct, using universal instantiation and modus tollens.

b This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.

c This is not correct. After applying universal instantiation, it contains the fallacy of affirming the conclusion.

d This is correct using universal instantiation and modus ponens.

6 Problem 1.5.48

The number 1 has this property, since the only positive integer not exceeding 1 is 1 itself, and therefore the sum is 1. This is a constructive proof.