

CS202 Midterm Exam

October 12th, 2005

Write your answers on the exam. Justify your answers. Work alone. Do not use any notes or books.

There are four problems on this exam, each worth 20 points, for a total of 80 points. You have approximately 50 minutes to complete this exam.

1 A recurrence (20 points)

Give a simple formula for $T(n)$, where:

$$T(0) = 1.$$

$$T(n) = 3T(n-1) + 2^n, \text{ when } n > 0.$$

2 An induction proof (20 points)

Prove by induction on n that $n! > 2^n$ for all integers $n \geq n_0$, where n_0 is an integer chosen to be as small as possible.

3 Some binomial coefficients (20 points)

Prove that $k\binom{n}{k} = n\binom{n-1}{k-1}$ when $1 \leq k \leq n$.

4 A probability problem (20 points)

Suppose you flip a fair coin n times, where $n \geq 1$. What is the probability of the event that both of the following hold: (a) the coin comes up heads at least once and (b) once it comes up heads, it never comes up tails on any later flip?