

CS202 Final Exam

December 19th, 2008

Write your answers in the blue book(s). Justify your answers. Work alone. Do not use any notes or books.

There are five problems on this exam, each worth 20 points, for a total of 100 points. You have approximately three hours to complete this exam.

1 Some logical sets (20 points)

Let A , B , and C be sets.

Prove or disprove: If, for all x , $x \in A \rightarrow (x \in B \rightarrow x \in C)$, then $A \cap B \subseteq C$.

2 Modularity (20 points)

Let m be an integer greater than or equal to 2. For each a in \mathbb{Z}_m , let $f_a : \mathbb{Z}_m \rightarrow \mathbb{Z}_m$ be the function defined by the rule $f_a(x) = ax$.

Show that f_a is a bijection if and only if $\gcd(a, m) = 1$.

3 Coin flipping (20 points)

Take a biased coin that comes up heads with probability p and flip it $2n$ times.

What is the probability that at some time during this experiment two consecutive coin-flips come up both heads or both tails?

4 A transitive graph (20 points)

Let G be a graph with n vertices on which the adjacency relation is transitive: whenever there is an edge uv and an edge vw , there is also an edge uw . Suppose further that G is connected. How many edges does G have?

5 A possible matrix identity (20 points)

Prove or disprove: If A and B are symmetric matrices of the same dimension, then $A^2 - B^2 = (A - B)(A + B)$.