CS202 Midterm Exam

October 24th, 2008

Write your answers on the exam. Justify your answers. Work alone. Do not use any notes or books.

There are four problems on this exam, each worth 20 points, for a total of 80 points. You have approximately 50 minutes to complete this exam.

1 Some sums (20 points)

Let $a_0, a_1, \ldots$ and $b_0, b_1, \ldots$ be sequences such that for i in $\mathbb{N}$, $a_i \leq b_i$.

Let $A_i = \sum_{j=0}^{i} a_j$ and let $B_i = \sum_{j=0}^{i} b_j$.

Prove or disprove: For all i in $\mathbb{N}$, $A_i \leq B_i$.

2 Nested ranks (20 points)

You are recruiting people for a secret organization, from a population of n possible recruits. Out of these n possible recruits, some subset M will be members. Out of this subset M, some further subset C will be members of the inner circle. Out of this subset C, some further subset X will be Exalted Grand High Maharajaraja Panjandrums of Indifference. It is possible that any or all of these sets will be empty.

If the roster of your organization gives the members of the sets M, C, and X, and if (as usual) order doesn't matter within the sets, how many different possible rosters can you have?

3 Nested sets (20 points)

Let A, B, and C be sets.

1. Prove or disprove: If $A \in B$, and $B \subseteq C$, then $A \subseteq C$.
2. Prove or disprove: If $A \subseteq B$, and $B \subseteq C$, then $A \subseteq C$.

4 An efficient grading method (20 points)

A test is graded on a scale of 0 to 80 points. Because the grading is completely random, your grade can be represented by a random variable X with $0 \leq X \leq 80$ and $\mathrm{E}[X] = 60$.

1. What is the maximum possible probability that $X = 80$?

2. Suppose that we change the bounds to $20 \leq X \leq 80$, but $\mathrm{E}[X]$ is still 60. Now what is the maximum possible probability that $X = 80$?