CS202 Midterm Exam

October 21st, 2010

Write your answers on the exam. Justify your answers. Work alone. Do not use any notes or books.

There are four problems on this exam, each worth 20 points, for a total of 80 points. You have approximately 75 minutes to complete this exam.

1 A partial order (20 points)

Let $S \subseteq \mathbb{N}$, and for any $x, y \in \mathbb{N}$, define $x \preceq y$ if and only if there exists $z \in S$ such that $x + z = y$.

Show that if $\preceq$ is a partial order, then (a) 0 is in S and (b) for any x, y in S, $x + y$ is in S.

2 Big exponents (20 points)

Let p be a prime, and let $0 \le a < p$. Show that $a^{2p-1} = a \pmod{p}$.

3 At the playground (20 points)

Let $L(x, y)$ represent the statement "x likes y" and let $T(x)$ represent the statement "x is tall," where x and y range over a universe consisting of all children on a playground. Let m be "Mary," one of the children.

1. Translate the following statement into predicate logic: "If x is tall, then Mary likes x if and only if x does not like x."

2. Show that if the previous statement holds, Mary is not tall.

4 Gauss strikes back (20 points)

Give a closed-form formula for $\sum_{k=a}^{b} k$, assuming $0 \le a \le b$.