1 A partial order (20 points)

Let $S \subseteq \mathbb{N}$, and for any $x, y \in \mathbb{N}$, define $x \preceq y$ if and only if there exists $z \in S$ such that $x + z = y$.

Show that if $\preceq$ is a partial order, then (a) 0 is in $S$ and (b) for any $x, y$ in $S$, $x + y$ is in $S$.

2 Big exponents (20 points)

Let $p$ be a prime, and let $0 \leq a < p$. Show that $a^{2p-1} = a \pmod{p}$.

3 At the playground (20 points)

Let $L(x, y)$ represent the statement “$x$ likes $y$” and let $T(x)$ represent the statement “$x$ is tall,” where $x$ and $y$ range over a universe consisting of all children on a playground. Let $m$ be “Mary,” one of the children.

1. Translate the following statement into predicate logic: “If $x$ is tall, then Mary likes $x$ if and only if $x$ does not like $x$.”

2. Show that if the previous statement holds, Mary is not tall.

4 Gauss strikes back (20 points)

Give a closed-form formula for $\sum_{k=a}^{b} k$, assuming $0 \leq a \leq b$. 

1