Commutativity Race Detection

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Abstract
This paper introduces the concept of a commutativity race. A commutativity race occurs in a given execution when two library method invocations can happen concurrently yet they do not commute. Commutativity races are an elegant concept enabling reasoning about concurrent interaction at the library interface.

We present a dynamic commutativity race detector. Our technique is based on a novel combination of vector clocks and a structural representation automatically obtained from a commutativity specification. Conceptually, our work can be seen as generalizing classical read-write race detection.

We also present a new logical fragment for specifying commutativity conditions. This fragment is expressive, yet guarantees a constant number of comparisons per method invocation rather than linear with unrestricted specifications.

We implemented our analyzer and evaluated it on real-world applications. Experimental results indicate that our analysis is practical: it discovered harmful commutativity races with overhead comparable to state-of-the-art, low-level race detectors.

1. Introduction
In response to the growing complexity of software, common patterns are increasingly being encapsulated into thread-safe libraries (e.g. Java collections). At the same time, virtually all modern applications use some form of concurrency and heavily rely on various libraries to accomplish their objective. This raises a key challenge: even though a library is implemented correctly, concurrent threads can invoke its operations in a way which causes undesirable interference at the library interface level, leading to incorrect program behaviors. We observe that such errors are fundamentally caused when concurrent threads perform non-commutative library operations. We refer to this phenomenon as a commutativity race. To illustrate the concept, consider the following simple program:

1: fork T2;
2: m.put(5,7);
3: int v=m.get(5);

Initially, the concurrent hashmap \( m \) has all keys initialized to the value 1. Here, we have a commutativity race between \( m\).put(5,7) and \( m\).get(5): the two operations do not commute and they can happen in any order. As a result of this commutativity race, the values returned by get differ in each of the two possible executions, returning 7 if \( m\).put(5,7) occurred before \( m\).get(5) in the execution or returning 1 otherwise. Such non-determinism can sometimes lead to undesirable behaviors further in the execution. In fact, in Java, if \( m \) was initially empty, and \( m\).get executes before \( m\).put(5,7), the program will throw a NullPointerException when unboxing the result from \( m\).get. Of course, not all commutativity races are harmful, however the presence of a commutativity race may indicate undesirable interference.

Commutativity race detection. In this paper, we introduce the concept of a commutativity race and present a dynamic commutativity race detector with the appropriate formal guarantees. Our approach is parametric on a declarative commutativity specification and can be used to analyze any library equipped with such a specification. The analysis is based on a novel combination of commutativity information with classic vector clocks [12, 14]. Conceptually, our work can be seen as a generalization of traditional data race detection (e.g. [3]) to deal with much richer and abstract notions of conflict, beyond the level of basic reads and writes.

We introduce an expressive logical fragment (ECL) for capturing commutativity conditions as well as an automatic translation procedure from ECL to a structural representation used by the commutativity race detector. ECL admits practical specifications (e.g. maps, sets) yet is amenable to efficient analysis: any commutativity condition expressed in ECL can be checked by the analysis via a constant number of comparisons per method invocation as opposed to linear with unrestricted specifications. We believe the results in this paper are of interest beyond race detection: the structural representation, ECL and the translation procedure between them can serve as a basis for other concurrency analyzers as well as for optimistic concurrency schemes (e.g. transactional memory).

Contributions. The main contributions of this paper are:

- A dynamic commutativity race detector based on a novel combination of classic vector clocks for tracking the happens-before relation with a structural representation of a commutativity specification (Section 5). Our approach generalizes traditional race detection to richer, more abstract notions of conflict.
- A logical fragment, called ECL, which captures useful commutativity conditions (e.g. maps), yet allows the analysis to perform a constant number of operations per method invocation, as opposed to linear with arbitrary specifications (Section 6.1).
- A translation procedure from ECL to the structural representation used by the analysis (Section 6.2).
- An implementation and an evaluation of our approach on two industrial Java applications (Section 7). Our results indicate that the performance overhead is acceptable and further, our tool discovered harmful commutativity races in these applications when using ConcurrentHashMap.
2. Overview

Next, we informally introduce and motivate the concepts underlying our approach. Full formal details are provided in later sections.

Consider the example program shown in Fig. 1. The program obtains a list of hostnames, creates a connection to each host and stores these connections in an initially empty associative map (a dictionary for which every key is initially associated with nil) such that every hostname is associated with a connection. In Java, the dictionary can be represented by ConcurrentHashMap. Connections are established in parallel as the program forks a thread for each connection. The command joinall waits for all previously created threads to finish. Once all threads have completed, the program prints the total number of established connections.

Example of a commutativity race. If hosts contains duplicates (e.g. hosts = ['a.com', 'a.com']), more than one thread will attempt to connect to the same host, leading to a commutativity race between a successful o.put invocation in one thread and an unsuccessful o.put operation in another. Such a commutativity race can indicate a potential application error: the developer did not think of the fact that the input list may contain duplicate hosts. Further, with every attempt to create a duplicate connection, there will be an additional short-lived connection object that is created and never used later, overburdening the underlying memory management system.

We next discuss our approach to detecting commutativity races. The approach consists of several ingredients. These ingredients as well as the flow between them are illustrated in Fig. 2. We discuss these ingredients in the context of our example. Later sections provide the appropriate formal treatment.

Step 1: obtain a commutativity specification. We capture commutativity between method invocations by using logical formulas: a natural, declarative way to capture commutativity [8]. That is, for every pair of methods of a given library, we provide a formula which describes when the two methods commute. For instance, the following formula describes when two put operations, put(k1, v1) / p1 and put(k2, v2) / p2, commute:

$$k_1 \neq k_2 \lor (v_1 = p_1 \land v_2 = p_2)$$

Step 2: obtain an access point representation. Next, our translator (discussed in Section 6.2) automatically converts a commutativity specification into what we call an access point representation, a form of an intermediate executable representation used by the analysis. Its main purpose is to reduce the number of checks that the analysis needs to perform. An access point representation captures the “micro actions” (called access points) that are relevant to commutativity checking. For instance, a successful o.put(k, v) / nil is translated into two access points ovck and o:resize denoting that: i) the value associated with the particular key has changed, and ii) the size of the dictionary o has changed. Conflict checking can then be performed on individual access points as opposed to on entire invocations (if the analysis had worked using the commutativity specification directly). More generally, working with the access point representation of certain commutativity specifications (which we discuss in Section 6) leads to better asymptotic complexity of the analysis algorithm (i.e. constant over linear).

Step 3: commutativity race detection. Finally, our online dynamic analyzer consumes a program trace and checks the trace for commutativity races. Consider an execution of our running example shown in Fig. 3. First, thread τ2 associates a connection c1 with ‘a.com’ in the dictionary. As the dictionary is initially empty, put returns nil. Next, thread τ2 associates a new connection object c2 with ‘a.com’, overwriting and returning the previous connection c1. Finally, the main thread waits for both threads to complete and obtains the size of the dictionary, which is 1.

In the figure, we show the information computed by our algorithm when analyzing the trace, namely the vector clocks which capture the temporal relationship between invocations (also called actions) as well as the access points which capture the commutativity relationship between invocations. Here, each invocation is inside a shaded area (top right names the action) annotated with a vector clock in the top-left corner. A commutativity race occurs when there are two unordered actions (i.e. their vector clocks are non-comparable), yet their access points conflict.

In our example, actions a1 and a2 participate in a commutativity race. The reason is that: i) both of these actions are unordered as their vector clocks are non-comparable (we compare vector clocks for races directly). More generally, working with the access point representation of certain commutativity specifications (which we discuss in Section 6) leads to better asymptotic complexity of the analysis algorithm (i.e. constant over linear).

In our example, actions a3 and a1 do not participate in a commutativity race because they are ordered: the vector clock (3, 0, 1) of action a3 is less than the vector clock (4, 1, 1) of action a1. Note that there had not been a joinall statement in the code, then the two actions would conflict because action a1 resizes the dictionary and touches access point o:resize which conflicts with the access point o:size touched by the action a3.
Finally, actions $a_3$ and $a_2$ do not participate in a commutativity race because they are ordered according to their vector clocks. Interestingly, there had not been a `joinall` statement in the code then these two actions would still not participate in a commutativity race because they touch access points which do not conflict: intuitively, action $a_2$ is not resizing the dictionary and hence cannot affect the result of $a_3$.

**Motivation for access points.** To give an intuition why we introduce access points, consider the example in Fig. 4. Here, in the top part, we have the main thread invoking `size` which conflicts with all three successful `put` invocations of other threads. An analysis which directly uses the logical specification would need to perform three checks for `size`: one with each `put` operation. However, for checking whether `size` and `put` commute, the only relevant information is whether a `put` operation is resizing the map. The bottom part of the figure shows the access points that our translation produces for each invocation. Here, each `o.put(k, v)/nil` invocation is translated into two access points: `o:resize` and `o:size`.

Interestingly, we see that each of the three put operations share the access point `o:resize` capturing the fact that the size of the dictionary changed. On the other hand, the `size()` operation generates a single access point `size`. With access points, a commutativity analysis now only needs to check whether the `size` access point conflicts with the resize access point `o:size`. Importantly, this is a single conflict check and not three conflict checks as would be necessary without using access points.

**Discussion.** Before we proceed further, we discuss three key points. First, the commutativity race detector is orthogonal to how the access point representation is obtained. This representation can be obtained manually or automatically by translating from a commutativity specification. Second, the access point representation can be used to generalize other concurrency analyzers to high-level commutativity notions of conflict (e.g., atomicity [5]), or to enable more general optimistic concurrency control schemes (e.g., [10]). Similarly, the translator from commutativity specifications to access point representations is useful in those (and potentially other) settings as well. Third, as we will see later, the asymptotic time complexity of the analysis presented in this work differs depending on the different access point representations. To cleanly capture this fact, in Section 6 we present a commutativity logical fragment and a translation of any formula in that fragment to an access point representation which enables the analysis to always lookup a constant number of access points instead of linear as would be needed with arbitrary access point representations.

### 3. Preliminaries

In this section we provide necessary definitions of several terms used later in the paper.

#### 3.1 Execution model

**Actions.** We consider concurrent programs consisting of threads that communicate via shared objects. Each object $o \in \text{Obj}$ can be in a set of possible abstract states. That is, we are interested in the abstract states of the object as described by its specification and not in the actual implementation details of the object. For example, the abstract state of a dictionary object is a key-value mapping of type $K \rightarrow (V \cup \{\text{nil}\})$ where $K$ is a set of possible keys, $V$ is the set of possible values and nil is a special no-value.

The shared state of a program consists of the abstract states of all shared objects. The space of possible shared states is denoted by $H$. For example, if a program references only one shared object, a dictionary, then $H = K \rightarrow (V \cup \{\text{nil}\})$. We assume that object methods are given by specifying their effects on the shared state.

#### Figure 5. Dictionary methods and their effects

<table>
<thead>
<tr>
<th>Method</th>
<th>Effect on the dictionary $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>put $(k, v)/p$</td>
<td>$d \rightarrow d' \iff d'(k \rightarrow v) = d[k \rightarrow v]$ and $p = d(k)$</td>
</tr>
<tr>
<td>get $(k)/v$</td>
<td>$d \rightarrow d' \iff d'(k) = d$ and $v = d(k)$</td>
</tr>
<tr>
<td>size()/$r$</td>
<td>$d \rightarrow d' \iff d'(r) = d$ and $r =</td>
</tr>
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the trace $\pi$ has a multiset\footnote{By a multiset we mean a set of labeled points rather than a set with element multiplicities.} of events $\{e_i = \tau_i : a_i\}_{i=1}^n$. Such a multiset is ordered by event position, that is $e_i \leq e_j$ iff $i \leq j$. The fact that $e$ is an event will be denoted by $e \in Evt$. If $e$ occurs in a trace $\pi$ we will write $e \in \pi$.

3.2 Happens-before

Happens-before relations\footnote{The notation $\varphi_{m_1}^{m_2}(\tilde{y}_1;\tilde{y}_2)$ is an alternative to $\varphi_{m_1}^{m_2}(\tilde{y}_1)/\tilde{r}_1$.} are a standard way to encode causality between events. A happens-before relation for a trace $\pi$ is a certain partial ordering $\preceq$ on $\pi$’s multiset of events. This order records causal relationships between the events. When $e_i \preceq e_j$ then there must be some cause that forces $e_i$ to appear before $e_j$. If neither $e_i \preceq e_j$ nor $e_j \preceq e_i$ we say that $e_i$ and $e_j$ may happen in parallel and denote it by $e_i \parallel e_j$. The relation $\preceq$ satisfies:

- if $e_i \preceq e_j$ then $e_i \leq e_j$
- if the program forces $e_i$ to occur before $e_j$ then $e_i \preceq e_j$

For example, one thread awaiting for the termination of another forces subsequent events from the waiting thread to occur after the events from the other thread. Another example is the sequential execution of actions by a single thread. In particular, if $e_i \leq e_j$ and $e_j$ have occurred in the same thread then $e_i \preceq e_j$. In other words, if $e_i \parallel e_j$ then $e_i$ and $e_j$ must have occurred in different threads.

Vector clocks. Vector clocks\footnote{An example of a commutativity specification for our dictionary object $o$ for the method pair $\text{put}(k_1,v_1)/p_1$ and $\text{put}(k_2,v_2)/p_2$ would be: $\varphi_{\text{put}}(k_1,v_1,r_1;k_2,v_2,r_2) := k_1 \neq k_2 \lor (v_1 = p_1 \land v_2 = p_2)$} are a well-known technique for computing the happens-before relation. A vector clock is a map $c \in \text{Tid} \to \mathbb{N}$ that records a timestamp $c(\tau)$ from the local clock of every thread $\tau$ for each $\tau \in \text{Tid}$. The set of vector clocks $\text{VC}$ is denoted by $VC$. The set of vector clocks $VC$ is ordered pointwise becoming a lattice with a smallest element. The additional operation $inc_v$ performs one timestamp increment of the $v$ component:

$$c_1 \preceq c_2 \iff c_1(\tau) \leq c_2(\tau) \text{ for all } \tau \in \text{Tid}$$

$$c_1 \parallel c_2 = \tau \Rightarrow \max(c_1(\tau),c_2(\tau))$$

$$\tau \mapsto 0$$

$$inc_v(c) = \tau \mapsto c(\tau) + 1 \text{ if } \tau = v \text{ else } c(\tau)$$

A happens-before relation $\preceq$ is represented by associating a vector $vc(e)$ clock with every event $e$

$$vc : Evt \to VC$$

in a way such that $e_i \preceq e_j$ iff $vc(e_i) \preceq vc(e_j)$. Informally, the vector clock $vc(e)$ reflects the dependency of $e$ on the progress of every event.

4. Commutativity Specifications and Their Representation

We next discuss how we capture commutativity specifications as well as how these commutativity specifications can be represented in a form that is usable by a dynamic analysis system. This representation is used later by our commutativity race detector in Section 5 and can potentially be applied to generalize other concurrency analyzers. Hence, the discussion in this section is also of interest beyond race detection.

4.1 Logical Specifications

The conditions under which two actions commute are often conveniently specified in the form of logical formulas. This allows formal treatment of various types of conditions. Briefly, a logical commutativity specification is a predicate $\varphi$ over pairs of actions $a,b \in Act$ such that $\varphi(a,b)$ implies that $a$ and $b$ commute. Commutativity specification of a dictionary object is given on Fig. 6.

$\varphi_{\text{put}}(k_1,v_1)/p_1 := k_1 \neq k_2 \lor (v_1 = p_1 \land v_2 = p_2)$

$\varphi_{\text{put}}(k_1,v_1)/p_1 := k_1 \neq k_2 \lor (v_1 = p_1)$

$\varphi_{\text{put}}(k_1,v_1)/p_1 := (v_1 = nil \land p_1 = nil) \lor (v_1 \neq nil \land p_1 \neq nil)$

$\varphi_{\text{put}}(k_1,v_1)/p_1 := (v_1 = nil \land p_1 = nil) \lor (v_1 \neq nil \land p_1 \neq nil)$

$\varphi_{\text{get}}(k_1,v_1)/p_1 := (v_1 = nil \land p_1 = nil) \lor (v_1 \neq nil \land p_1 \neq nil)$

$\varphi_{\text{get}}(k_1,v_1)/p_1 := (v_1 = nil \land p_1 = nil) \lor (v_1 \neq nil \land p_1 \neq nil)$

$\varphi_{\text{size}}(r_1)/r_1 := \text{true}$

$\varphi_{\text{size}}(r_1)/r_1 := \text{true}$

$\varphi_{\text{size}}(r_1)/r_1 := \text{true}$

Figure 6. Commutativity specification of a dictionary object

The kind of commutativity specifications we discuss in this work are those that can be expressed without mentioning the object state. A more relaxed notion of commutativity that can depend on object state may be useful, however this may have significant implications on its mechanized checking and we leave those as future work. It is worth mentioning that in certain cases exposing hidden state as shadow return values may allow obtaining more precise specification. However, this cannot capture the general notion of “two actions commute at a specific state”.

Next, we proceed to define logical commutativity specifications, soundness of such specifications and commutativity races.

Definition 4.1. Given a suitable logic, a logical commutativity specification for a pair of methods $m_1,m_2$ of the same object is given by a logical formula $\varphi_{m_1}^{m_2}$ with its free variables collected into the list $\langle \tilde{x}_1;\tilde{x}_2 \rangle$ so the variables $\tilde{x}_i$ match the arguments and returns of $m_i$. A logical commutativity specification $\Phi$ for an object with methods $M$ is a set of such specifications $\varphi_{m_1}^{m_2}(\tilde{x}_1;\tilde{x}_2)$ for each $\langle m_1,m_2 \rangle \subseteq M$.

For a method specification $\varphi_{m_1}^{m_2}(\tilde{x}_1;\tilde{x}_2)$ and suitable terms $\tilde{t}_1,\tilde{t}_2$ we will write $\varphi_{m_1}^{m_2}(\tilde{t}_1;\tilde{t}_2)$ for the substitution of $\tilde{t}_1$ for $\tilde{x}_1$ into the formula $\varphi_{m_1}^{m_2}(\tilde{x}_1;\tilde{x}_2)$. Similarly, we can evaluate the formula for concrete values $u_1,u_2$. We require specifications to represent symmetric predicates on actions, that is we require that $\varphi_{m_1}^{m_2}(\tilde{x}_1;\tilde{x}_2)$ is logically equivalent to $\varphi_{m_2}^{m_1}(\tilde{x}_1;\tilde{x}_2)$.

Example. An example of a commutativity specification for our dictionary object $o$ for the method pair $\text{put}(k_1,v_1)/p_1$ and $\text{put}(k_2,v_2)/p_2$ would:

$$\varphi_{\text{put}}(k_1,v_1,r_1;k_2,v_2,r_2) := k_1 \neq k_2 \lor (v_1 = p_1 \land v_2 = p_2)$$

Note that when we swap parameters we get the equivalent formula:

$$\varphi_{\text{put}}(k_2,v_2,r_2;k_1,v_1,r_1) := k_2 \neq k_1 \lor (v_2 = p_2 \land v_1 = p_1)$$

For a specification $\varphi_{m_1}^{m_2}(\tilde{x}_1;\tilde{x}_2)$ and two actions $a = o.m_1(\tilde{u}_1)/\tilde{v}_1$ and $b = o.m_2(\tilde{u}_2)/\tilde{v}_2$, we write $\varphi_{m_1}^{m_2}(a,b)$ for the evaluation $\varphi_{m_1}^{m_2}((u_1\tilde{v}_1);(u_2\tilde{v}_2))$.

We would expect commutativity specifications to actually say something about the commutativity properties of the object in question. Sound specifications imply commutativity.

Definition 4.2. A logical commutativity specification $\Phi$ for some object is sound iff for every method specification $\varphi_{m_1}^{m_2}(\tilde{x}_1;\tilde{x}_2) \in \Phi$ and two actions $a,b \in m_1,m_2$ respectively, we have that $\varphi_{m_1}^{m_2}(a,b)$ implies $a \times b$.

Note that the above definition allows for imprecise commutativity specifications. Even though actions commute, the specification is allowed to say that they do not. One could replace the implication with an if-and-only-if condition but this is unnecessarily for our algorithm. We are now ready to define the concept of a commutativity race, a central concept of our paper.
\[ X_o = \{ o;k \}_{k \in K} \cup \{ o:w;k \}_{k \in K} \cup \{ o:size,o:resize \} \]

(a) Access points

<table>
<thead>
<tr>
<th>Action Type</th>
<th>Access Points ((\eta_0))</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>o:put ((k,v)/p)</td>
<td>o:w;k,o:resize</td>
<td>( v \neq p ) and size changed</td>
</tr>
<tr>
<td>o:w;k</td>
<td>v ( \neq p ) and size unchanged</td>
<td></td>
</tr>
<tr>
<td>o:resize</td>
<td>v = p</td>
<td></td>
</tr>
<tr>
<td>o:get ((k)/v)</td>
<td>o:resize</td>
<td></td>
</tr>
<tr>
<td>o:size</td>
<td>o:size</td>
<td></td>
</tr>
</tbody>
</table>

(b) Action access points

<table>
<thead>
<tr>
<th>( C_o )</th>
<th>o:resize</th>
<th>o:size</th>
</tr>
</thead>
<tbody>
<tr>
<td>o:w;k</td>
<td>( k = l )</td>
<td>yes</td>
</tr>
<tr>
<td>o:resize</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>o:size</td>
<td>yes</td>
<td></td>
</tr>
</tbody>
</table>

(c) Conflict relation \( C_o \)

Figure 7. Access point representation for a dictionary.

Definition 4.3. For a trace \( \pi \), a pair of events \( e_i, e_j \in \pi \) participate in a commutativity race with respect to a partial order \( \preceq \) and a commutativity specification of their corresponding methods \( \varphi_{m_1}^o(\vec{x}_1; \vec{x}_2) \) iff \( e_i \parallel e_j \) (the events may happen in parallel) and their corresponding actions \( a, b \) are not specified to commute, that is, \( \varphi_{m_1}^o(a, b) \) does not hold.

Note that if the commutativity specification is imprecise, it is possible to report a commutativity race even if actions do commute. To avoid clutter, we use the term commutativity race regardless of whether the specification is precise or not.

4.2 Access point representation

We next introduce a structure which is useful for capturing commutativity specifications in a way that can be used by a dynamic program analyzer. We refer to these structures as access point representations (of the object’s commutativity properties).

Definition 4.4. An access point representation for an object \( o \in \text{Obj} \) is a tuple \((X_o, \eta_o, C_o)\), where:

1. \( X_o \) is a set of access points.
2. \( \eta_o : \text{Act}_o \rightarrow \mathcal{P}(X_o) \) indicates the finite set of access points touched by each action of the object (\( \text{Act}_o \) stands for the set of all actions of object \( o \)).
3. \( C_o \subseteq X_o \times X_o \) is a symmetric binary relation describing which access points conflict.

Example: Dictionary

Fig. 7 presents an access point representation for a dictionary object \( o \). Fig. 7(a) shows two groups of access points. The first group contains two access points for every possible key: \( o:resize \) indicates that the value for key \( k \) has been read; \( o:w;k \) indicates that the value for key \( k \) has changed. The second group has only two access points: \( o:size \) indicates that the dictionary size has been observed; \( o:resize \) indicates that the dictionary has been resized.

Fig. 7(b) shows the possible types of actions and the access points which are touched upon the execution of that action. For example, the action \( o:put(5,1)/nil \) changes the value for key 5 and also changes the size of the dictionary. Therefore, \( \eta_o(o:put(5,1)/nil) = \{ o:w:5, o:resize \} \). Whether the action \( o:put(k,v)/p \) changes the size is determined by the parameters alone: either \( v \neq \text{nil} \) and \( p = \text{nil} \) or \( v = \text{nil} \) and \( p \neq \text{nil} \).

Fig. 7(c) shows the conflict relation for each of the two groups of access points (for convenience split in two matrices). The entry in the matrix denotes when the two access points conflict. For example, \( o:w:5 \) and \( o:w:6 \) conflict (as \( k = l \)) while \( o:w:5 \) and \( o:w:6 \) do not conflict (as \( k \neq l \)). There are no conflicts between actions from different groups.

We now define what it means for an access point representation to precisely match a logical specification. In Section 6, we show a translation from a logical formula to an equivalent access point representation.

Definition 4.5. We say that \((X_o, \eta_o, C_o)\) represents a logical commutativity specification \( \Phi \) of \( o \) iff for all \( \varphi_{m_2}^o \in \Phi \) and all actions \( a \) of \( m_1 \) and \( b \) of \( m_2 \) we have:

\[ (\eta_o(a) \times \eta_o(b)) \cap C_o = \emptyset \iff \varphi_{m_2}^o(a, b) \]

Therefore, instead of a given logical specification \( \Phi \), we can now work with an access point representation \((X_o, \eta_o, C_o)\) equivalent to \( \Phi \). We utilize access point representations when detecting commutativity races.

5. Commutativity Race Detection

We next present our commutativity race detector, which is parameterized over an access point representation. We first motivate our approach, then we present our algorithm and discuss how its asymptotic complexity can be affected by choosing different commutativity specifications, and finally provide the formal guarantees.

5.1 Challenges

Recall that in Section 2, we discussed the benefits of working with an access point representation over working directly with a commutativity specification. Let us again consider the direct approach where we work with the commutativity specification directly. This approach works as follows. It records every action occurring in an execution and every time it encounters a new action, it checks whether there is a previously observed action on the same object that can happen concurrently with the current one and which does not commute with it.

Checking whether two actions \( a \) and \( b \) commute is done by simply checking whether the pair of actions provide a satisfying assignment to the corresponding commutativity formula. That is, whether \( \varphi_{m_1}^o(a, b) \) evaluates to true where \( m_1 \) is the method of action \( a \) and \( m_2 \) is the method of action \( b \).

A key challenge with the direct approach is that it is oblivious to the type of commutativity specification being checked and ignores the fact that many actions share commonalities. For instance, as illustrated earlier in Fig. 4, several \texttt{put} actions share the common access point \texttt{o:resize}. As a result, the algorithm records each action independently and requires \( \Theta(|A|) \) commutativity checks for each encountered action, where \( A \) is the set of all actions occurring in the execution.

Our algorithm aims to remedy these deficiencies by exploiting the structure present in commutativity specifications. Instead of working directly with logical formulas, the algorithm leverages access point representations (described in Section 4.2). Access point representations allow us to expose some of the common structure present in commutativity predicates (we already illustrated this earlier in Fig. 4).

Further, in Section 6, we will introduce an expressive logical fragment (called ECL) where formulas in that fragment are translated to access point representations that require only \( \Theta(1) \) checks for each encountered action as opposed to \( \Theta(|A|) \).
the standard way of updating vector clocks for the statements:

<table>
<thead>
<tr>
<th>Previous work</th>
<th>Modifications of auxiliary state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = \tau: \text{fork}(u)$</td>
<td>$T(u) \leftarrow \text{inc}_a(T(\tau))$</td>
</tr>
<tr>
<td>$e = \tau: \text{join}(u)$</td>
<td>$T(\tau) \leftarrow \text{inc}_r(T(\tau))$</td>
</tr>
<tr>
<td>$e = \tau: \text{acq}(l)$</td>
<td>$T(\tau) \leftarrow T(\tau) \cup L(l)$</td>
</tr>
<tr>
<td>$e = \tau: \text{rel}(l)$</td>
<td>$L(l) \leftarrow T(\tau)$</td>
</tr>
<tr>
<td></td>
<td>$T(\tau) \leftarrow \text{inc}_r(T(\tau))$</td>
</tr>
</tbody>
</table>

Our work

| $e = \tau: o.m(\vec{x})/\vec{y}$ | $\text{vc}(e) \leftarrow T(\tau)$ |
| Execute Algorithm 1 |

### Table 1. Handling synchronization and action events.

#### 5.2 Common Setting

Our approach is applicable to any parallel language. However, to convey an end-to-end picture we describe how our algorithm fits with the fork-join constructs. The first four rows in Table 1 describe the standard way of updating vector clocks for the statements:

- $\tau: \text{fork}(u)$ – thread $\tau$ creates thread $u \in \text{Tid}$.
- $\tau: \text{join}(u)$ – thread $\tau$ waits until thread $u$ terminates.
- $\tau: \text{acq}(l)$ – thread $\tau$ acquires a lock $l \in \text{Lock}$.
- $\tau: \text{rel}(l)$ – thread $\tau$ releases a lock $l \in \text{Lock}$.

To capture the happens-before relation, we maintain two auxiliary mappings $T: \text{Tid} \rightarrow \text{VC}$ and $L: \text{Lock} \rightarrow \text{VC}$. All vector clocks are initialized with $\bot$ and the auxiliary state is updated at every synchronization event in the trace according to Table 1. Such handling of vector clocks is standard (e.g. see [3]). The novel part of our approach, shown in the last row, is in handling of actions, specifically Algorithm 1 which we discuss next.

#### 5.3 Our Approach

Next, we present our algorithm for detecting commutativity races given an access point representation. The algorithm works on-the-fly and reports races as the trace is explored. The novel part in our dynamic analysis is handling of action events $e = \tau: o.m(\vec{x})/\vec{y}$ and is described in Algorithm 1 (in the algorithm we describe the handling of a single action event). We assume an access point representation $(X_o, \eta_o, C_o)$ for each object $o \in \text{Obj}$ used by the program. Moreover, we assume that the sets $X_o$ for different objects are disjoint. Let $\mathcal{X}$ be their union. Also, let $C_o(pt)$ denote all the access points that conflict with $pt$, that is, $C_o(pt) = \{pt' \mid (pt, pt') \in C_o\}$. The algorithm maintains the following additional auxiliary state:

- $ptvc: \mathcal{X} \rightarrow \text{VC}$. Here, we keep a vector clock for each access point. For an access point $pt$, we use $ptvc$ as a shortcut for $ptvc(pt)$. The entries $ptvc$ are initialized by the algorithm on demand.
- $active: \text{Obj} \rightarrow \mathcal{P}(\mathcal{X})$. Here, for each object, we keep the set of access points touched by all object actions performed so far, so $active(o)$ is a finite subset of $\mathcal{X}_o$. Initially, $active(o)$ is the empty set for every object $o$.

In the first phase (Line 2 to Line 7), the algorithm checks for a commutativity race. The algorithm iterates over each access point $pt$ touched by the action of the event $e$ and seeks all active points $pt' \in active(o)$ which conflict with $pt$. This is done by computing the intersection of $active(o)$ and $C_o(pt)$. A commutativity race is detected if a conflicting access point $pt'$ can be touched concurrently to the current event $e$ as determined by the vector clocks $\text{vc}(e)$ and $pt'.vc$.

In the second phase (Line 8 to Line 16), the vector clocks of the access points associated with the action of the event $e$ are updated. The vector clock of every such access point $pt$ accumulates the vector clocks of all actions that have touched $pt$ so far. If an access point $pt$ touched by the action $e$ was not active, it is made active.

#### Algorithm 1: Commutativity Race Detector

**Input:** event $e = \tau: o.m(\vec{x})/\vec{y}$, vector clock $\text{vc}(e)$, access point representation $(X_o, \eta_o, C_o)$

**Output:** report a commutativity race or update auxiliary state

```
begin
  // phase 1: check for commutativity races
  for $pt \in \eta_o(o.m(\vec{x})/\vec{y})$ do
    for $pt' \in active(o) \cap C_o(pt)$ do
      if $pt'.vc \not\subseteq \text{vc}(e)$ then
        report "commutativity race";
      end for / phase 2: update auxiliary state
  end for
  if $pt \in active(o)$ then
    $ptvc \leftarrow ptvc \cup \text{vc}(e)$
  else
    // initialize
    $ptvc \leftarrow \text{vc}(e)$;
    active(o) \leftarrow active(o) \cup \{pt\};
  end if
end
```

In the algorithm, the set $active(o)$ grows continuously. To reduce its size, several optimizations are possible. For example, if an object $o$ is reclaimed (i.e. collected, released, etc.), all of its access points and their corresponding vector clocks can be reclaimed too, as no new races can be reported on a dead object. In our tool, we have implemented this optimization by attaching the auxiliary state directly to the object to which it belongs and then removing that state whenever the object dies. Another conceptually interesting optimization that we leave for future work is to remove unnecessary active access points by exploiting properties of the access point representation $(X_o, \eta_o, C_o)$.

**Example.** Consider again the example trace from Fig. 3 discussed in Section 2 (repeated here for convenience):

<table>
<thead>
<tr>
<th>event action</th>
<th>identifier</th>
<th>vector clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_\tau: o.\text{put}(\langle a.\text{com}, c_1 \rangle)/nil$</td>
<td>$a_1$</td>
<td>$\langle 3, 0, 1 \rangle$</td>
</tr>
<tr>
<td>$t_\tau: o.\text{put}(\langle a.\text{com}', c_2 \rangle)/c_1$</td>
<td>$a_2$</td>
<td>$\langle 2, 1, 0 \rangle$</td>
</tr>
<tr>
<td>$t_m: \text{join}(\langle t_\tau, t_\tau \rangle)$</td>
<td>$a_3$</td>
<td>$\langle 4, 1, 1 \rangle$</td>
</tr>
</tbody>
</table>

Let us explain how Algorithm 1 detects the commutativity race from Fig. 3. On execution of action $a_1$, the algorithm checks that access point $pt = o.\text{put}(\langle a.\text{com}, c_1 \rangle)/nil$ does not conflict with any previous access point. This is true, as $pt$ is the first encountered access point. Then, at Line 14, the algorithm records that $pt$ was touched with vector clock $\langle 3, 0, 1 \rangle$. Next, action $a_2$ executes and touches the same access point $pt$. At Line 6, the algorithm encounters $pt$ again which according to $C_o$ conflicts with itself (the conflict matrix $C_o$ for dictionary is shown in Fig. 7(c)). Further, the vector clock of the previous access is $\langle 3, 0, 1 \rangle$ which is incomparable with $\langle 2, 1, 0 \rangle$ – the vector clock of the current event. Therefore, a
commutativity race is reported. The race reveals that action $a_2$ can execute concurrently with some previous action and both actions touch access points that conflict. Continuing, the algorithm updates the vector clock of $pt$ to the join $(3, 1, 1)$. In the remainder of the execution, no more commutativity races will be reported, because the vector clock of $a_2$ is $(4, 1, 1)$, which is greater than or equal to all previously recorded vector clocks.

5.4 Algorithm Complexity

The complexity of the algorithm is dominated by the number of iterations at Line 4. In particular, for an unrestricted access point representation, the set $C_o(pt)$ can be infinite. For example, a naive access point representation for a dictionary object can be obtained by associating an access point with each possible action. Then, the access point for the size action would conflict with an infinite number of put actions that change the dictionary size. In such cases, enumerating the intersection $active(o) \cap C_o(pt)$ at Line 4 must be accomplished by first enumerating the finite set $active(o)$ and then checking if every element belongs to $C_o(pt)$. That is, the time needed to process each action is at least linear in $|active(o)|$ and the set $active(o)$ grows as the program executes.

However, for certain specifications, we can obtain a $C_o(pt)$ that is finite, e.g., the dictionary specification in Fig. 6 has $C_o(pt)$ of size at most 2 (see Fig. 7(c)). In this case, the enumeration at Line 4 may be done the other way around: for every element of $C_o(pt)$, perform the constant time check whether it belongs to the set $active(o)$. Effectively, in this case, the number of iterations at Line 4 is bounded by $|C_o(pt)|$. Later, in Section 6 we present a class of specifications which also includes the dictionary specification in Fig. 6. For every specification in this class there is an upper bound on $|C_o(pt)|$ for any $pt \in X_o$, thus enabling better complexity of the commutativity race detection algorithm.

5.5 Guarantees

We next state two important guarantees which hold in our setting. Proof of the first theorem can be found in the appendix, while the second theorem is standard and we do not provide further discussion. The theorems are valid under the assumption that the only source of nondeterminism in the underlying system is due to nondeterministic scheduling.

Theorem 5.1. Algorithm 1 reports a commutativity race if and only if the observed trace $\pi$ contains a commutativity race.

The above theorem states that our analysis is precise in the sense that some commutativity race is reported if and only if some commutativity race exists according to the commutativity specification. In general, the first reported race is guaranteed to be a true race. The next theorem is independent of the algorithm and extrapolates commutativity race freedom to a class of traces.

Theorem 5.2. If a trace $\pi$ with a happens-before relation $\preceq$ has no commutativity races with respect to $\preceq$ and a sound commutativity specification $\Phi$, then all traces which admit $\preceq$ and start in the same state as $\pi$:

1. end in the same state as $\pi$, and
2. have no commutativity races with respect to $\preceq$ and $\Phi$

The theorem states that if an observed trace is free of races, then each trace which admits the same happens-before relation as the observed trace is free of races and computes the same end result (guarantees determinism for the initial state of $\pi$). This is practically useful as it means that there is no need to explicitly enumerate and check those traces with our algorithm.

6. Commutativity Logic and its Translation

In this section we introduce a logical fragment for describing commutativity specifications, called ECL, as well as a translation procedure from that fragment to access point representations. The key benefit of ECL over arbitrary commutativity specifications is that we can translate ECL formulas to access point representations in which an access point always conflicts with a bounded number of other access points. This has complexity implications on the analysis (see Section 5.4).

Why ECL? Our approach to defining ECL is inspired by the work of Kulkarni et al. [10] which characterized the SIMPLE class of formulas and proved that under certain restrictions only such formulas can be checked with a mechanism called abstract locks. Unfortunately, SIMPLE does not precisely capture many practically useful specifications such as sets or the dictionary example in Fig. 6. We observe that in the setting of dynamic race detection (and concurrency analysis in general), the restriction considered in Kulkarni et al.’s work is unnecessary. Therefore, our objective is to develop a logical fragment which improves the asymptotic complexity of commutativity race detection yet is expressive enough to capture practical commutativity specifications.

6.1 ECL: extended commutativity logic

We next describe our logical fragment. We need to distinguish between arguments coming from the two actions referred in a specification formula. For this reason let $V_1$ and $V_2$ be two disjoint supplies of variables intended for arguments to the first method and for arguments to the second method respectively. We assume that the variables range over some domain $U$.

Because ECL extends SIMPLE, for convenience, we first repeat the definition of SIMPLE from Kulkarni et al.:

Definition 6.1. $L_S$

$S ::= V_1 \neq V_2 \mid S \land S \mid true \mid false$

As mentioned, $L_S$ does not allow one to capture many practical specifications. Therefore, in order to allow more expressive commutativity specifications, we combine $L_S$ with an additional fragment $L_B$ of more complex structure. Mainly, $L_B$ allows one to express constraints other than $x \neq y$.

Let $P_V$ stand for a set of atomic formulas interpreted in $U$ with their variables restricted to be in a set $V$. In addition to equality, $x = y$, the set $P_V$ may also contain formulas such as $x < y$, $x \cdot y = z$, etc. The sets $P_{V_1}$, $P_{V_2}$ are a parameter to the following:

Definition 6.2. $L_B$

$B ::= P_{V_1} \mid P_{V_2} \mid \neg B \mid B \land B \mid B \lor B \mid true \mid false$

The key restriction on atomic $L_B$ formulas is that their arguments must come from only one of $V_1$ or $V_2$. Therefore, the main characteristic property of $L_B$ is that the truth value of any atomic $L_B$ subformula depends on the valuation of variables in either $V_1$ or in $V_2$, but not in both.

Example. Suppose that $V_1 = \{x, y\}$ and $V_2 = \{z\}$. Then, $L_B$ allows the atomic formulas $x < y$ and $0 < z$, but not the atomic formula $x < z$, because this atomic formula contains variables from both $V_1$ and $V_2$. $L_B$ also allows us to capture the formula $x < y \land 0 < z$. Here, the truth value of the atomic sub-formula $x < y$ depends only on the variables in $V_1$. Similarly, the truth value of the atomic sub-formula $0 < z$ depends only on the variables in $V_2$.

The extended commutativity logic (ECL) combines the two fragments $L_S$ and $L_B$. ECL combines these by permitting conjunctions of ECL formulas and disjunctions of ECL and $L_B$ formulas.
Definition 6.3. ECL

$$X ::= S | B | X \land X | X \lor B$$

For an ECL method specification \( \varphi_m(x; y) \) the variables \( x \) must be drawn from the set \( V_1 \), while the variables \( y \) must be drawn from the set \( V_2 \). A key property of ECL, easily proved by structural induction, is the following:

Lemma 6.4. For any ECL formula, if a truth assignment to all the \( L_B \) atomic subformulas is given, then the whole formula simplifies to a \( L_S \) formula.

Example. Here is an example commutativity specification expressible in ECL, also shown earlier in Fig. 6:

$$\varphi_{\text{REP}}(k_1, v_1, p_1; k_2, v_2, p_2) := k_1 \neq k_2 \lor v_1 = p_1 \land v_2 = p_2$$

Here \( k_1, v_1, p_1 \in V_1 \) and \( k_2, v_2, p_2 \in V_2 \). This formula is not expressible in \( \text{SIMPLE} \) for two reasons: first, the formula contains a disjunction and second, the formula compares for equality, i.e. \( v_1 = p_1 \).

6.2 Translation from ECL to an access point representation

Recall that a logical commutativity specification \( \Phi \) for an object \( o \in \\text{Obj} \) consists of a commutativity specification \( \varphi_{\text{comm}} \) for each pair of methods \( m_1, m_2 \). Next, we describe how to obtain an access point representation for such a specification \( \Phi \) in ECL. This conversion of the ECL fragment builds upon, formalizes and extends the conversion of \( \text{SIMPLE} \) in [10].

Define \( \chi_o \). Access points are touched by methods during invocation. Each access point witnesses specific information about the invocation. The \( \beta \) component (to be defined below) collects the truth value of certain predicates over the action parameters and the return values. The translation defines two types of access points:

- \( o.m;\beta:ds \) simply witnesses that a method \( m \) of the object \( o \) has been invoked
- \( o.m;\beta:i:w_i \) witnesses the \( i \)-th argument or return value \( w_i \) of an invocation of \( m \).

The \( ds \) symbol is just a tag for the first type of access points. Each \( \beta \) vector maps an atomic \( L_B \) subformula from \( \Phi \) to a truth value. Let \( B(\Phi) \) be the set of all atomic formulas from the \( L_B \) fragment that occur in the specification \( \Phi \). Further, let each formula in \( B(\Phi) \) be normalized by dropping the distinction between the two types of variables \( V_1 \) and \( V_2 \).

Example. Consider the dictionary specification in Fig. 6. Its \( L_B \) atomic subformulas are \( v_1 = p_1, v_2 = p_2, v_1 = \text{nil}, p_1 = \text{nil}. \) To normalize, we just erase the indices obtaining that \( B(\Phi) \) is the set \( \{v = p, v = \text{nil}, p = \text{nil}\} \).

To define \( \chi_o \), let \( M \) be the set of methods of the object \( o \) and let \( \mathcal{U} \) be the domain of possible arguments and return values for these methods. The set of access points is the product:

$$\chi_o = \{o\} \times M \times (B(\Phi) \rightarrow \{\text{true}, \text{false}\}) \times (\{ds\} \cup \mathbb{N} \times \mathcal{U})$$

Define \( \beta \). Next, we define the \( \beta \) component associated with each action \( a = o.m(\vec{u})/\vec{v} \). Let \( B(\Phi, m) \) be the subset of \( B(\Phi) \) consisting of those atomic formulas that are relevant to the method \( m \), that is, normalized atomic subformulas of some \( \varphi_{m_2} \in \Phi \) with \( m \in \{m_1, m_2\} \). The vector \( \beta: B(\Phi, m) \rightarrow \{\text{true}, \text{false}\} \) collects the truth values of \( B(\Phi, m) \) evaluated on \( a = o.m(\vec{u})/\vec{v} \), that is, for \( q \in B(\Phi, m) \):

$$\beta(q) = q(\vec{u} \vec{v})$$

Example. Consider the action \( a = o.\text{put}(5, 6)/\text{nil} \). The relevant specifications to put are \( \varphi_{\text{put}}^{out}, \varphi_{\text{put}}^{out}, \varphi_{\text{size}} \) from Fig. 6. Thus, the relevant atomic formulas \( B(\Phi, \text{put}) \) are:

$$v = p, v = \text{nil}, p = \text{nil}$$

and substituting the action parameters \( k = 5, v = 6 \) and \( p = \text{nil} \) we obtain the vector:

$$\beta = \{(v = p) \rightarrow \text{false}, (v = \text{nil}) \rightarrow \text{false}, (p = \text{nil}) \rightarrow \text{true}\}$$

Define \( \eta_o \). Now we define which access points are touched by an action \( a = o.m(\vec{u})/\vec{v} \). Let us number the arguments and return values of \( a \), say by setting \( w_1, \ldots, w_n = \vec{u} \vec{v} \). Also, let us have the \( \beta \) vector associated with \( a \). The action touches the access points:

$$\eta_o(a) = \{o.m;\beta:ds\} \cup \{o.m;\beta:i:w_i | i = 1..n\}$$

Example. Let \( m_1 = \text{put}, \beta_1 \) map the atomic formula \( k = v \) to \( \text{false} \) and \( m_2 = \text{get} \). As \( B(\Phi, \text{get}) = \emptyset \) we have that \( \beta_2 = \emptyset \). Then, by substitution, we obtain:

$$\varphi_{\text{put}}^{out}[\beta_1; \beta_2] \equiv k_1 \neq k_2 \lor \text{false}$$

Define \( C_o \). Finally, we obtain the conflict relation \( C_o \). Define \( C_o \) as the symmetric closure of the relation \( R \) described next. For every \( \varphi_{m_2} \in \Phi \) we set:

1. \( (o.m_1;\beta_1; ds, o.m_2;\beta_2; ds) \in R \) iff \( \varphi_{m_2}^{out} [\beta_1; \beta_2] \equiv \text{false} \)
2. \( (o.m_1;\beta_1; i, u, o.m_2;\beta_2; j, v) \in R \) iff \( \varphi_{m_2}^{out} [\beta_1; \beta_2] \not\equiv \text{false} \) and contains a conjunct \( x_i \neq y_j \)
3. \( (pt_1, pt_2) \notin R \) for all other access points \( pt_1, pt_2 \)

Example. For the methods \( \text{put} \) and \( \text{get} \) in Fig. 6 we obtain the conflicts:

$$\{ o.\text{put};\beta_1:1, u, o.\text{get};\beta_2:1, v \} \in C_o \text{ iff } u = v \land \neg \beta_1(k = v)$$

Optimizations. Several additional optimizations can be applied to reduce the number of access points and obtain the representation shown in Fig. 7. The steps are worked out as an example in the appendix. It is important to note that the translation above preserves the mentioned \( \Theta(1) \) bound while the optimization steps further reduce the number of access points.

6.3 Equivalence and Complexity

The described translation from ECL specifications to access point representations is correct in the sense of preserving equivalence (cf. Definition 4.5).
Theorem 6.5. The access point representation obtained by translating an ECL formula $\Phi$ is equivalent to $\Phi$.

Now we consider the complexity of the obtained access point representation. For each method there are a finite number of possible $\beta$ vectors because specifications are finite and therefore there are only a finite number of atomic $L_B$ subformulas appearing. Also, if $o.m_1;\beta_1:i.u$ conflicts with any $o.m_2;\beta_2:j.v$ then $v$ must equal $u$, that is, the number of such conflicts is a function of the number of conjuncts in the specification. We obtain:

Theorem 6.6. In the resulting translation shown above, each access point conflict with a bounded number of other access points.

The bound depends on the size of the logical specification. This property is why we are able to obtain good asymptotic complexity of the race detection algorithm (see Section 5.4).

7. Evaluation

In this section, we present a preliminary evaluation of our approach. We focus on answering the following two questions: (i) is the overhead imposed by the commutativity race detector tolerable in practice as compared to state of the art low-level race detectors? (ii) is commutativity race detection effective for finding bugs?

To answer these questions, we implemented our commutativity analysis in a tool called RD2. RD2 is implemented in RoadRunner [4], an extensible dynamic analysis framework for Java. Then, we evaluated against two industrial Java applications and corresponding benchmarks, described below.

**H2 database server.** H2 is an open-source JDBC SQL database server. H2 recently added a Multi-Version Store (MVStore) which permits read operations to examine older versions of the data (i.e. Snapshot Isolation). The implementation is built upon several ConcurrentHashMap, which we instrumented. We used H2 version 1.3.174 and applied the Pole Position open source benchmark against the database. Pole Position is a framework that compares how different SQL databases compare against a suite of benchmark scenarios, called "circuits". We ran five "circuits" of the benchmark, three of which are focused on concurrency: ComplexConcurrency, QueryCentricConcurrency, InsertCentricConcurrency and two of which do not involve concurrent queries: Complex and NestedLists. We ran each of the benchmarks for two minutes and calculated the number of queries per second (qps) that the H2 database handles.

**Cassandra 2.0.** The Apache Cassandra database is an open-source distributed database. Cassandra maintains a performance rank of the database nodes. Its component DynamicEndpointSnitch calculates this rank by continuously accumulating statistics from the various nodes using ConcurrentHashMap. In this work, we ran our analysis against the DynamicEndpointSnitch test case which simulates dynamically changing node latencies.

<table>
<thead>
<tr>
<th>Application</th>
<th>Benchmark</th>
<th>Performance: qps or seconds</th>
<th>Races: total (distinct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H2 database</td>
<td>ComplexConcurrency</td>
<td>2011 qps</td>
<td>1784 (26)</td>
</tr>
<tr>
<td>H2 database</td>
<td>ComplexConcurrency (alternate query distrib.)</td>
<td>1610 qps</td>
<td>1121 (24)</td>
</tr>
<tr>
<td>H2 database</td>
<td>QueryCentricConcurrency</td>
<td>1666 qps</td>
<td>209 (4)</td>
</tr>
<tr>
<td>H2 database</td>
<td>InsertCentricConcurrency</td>
<td>1912 qps</td>
<td>1551 (25)</td>
</tr>
<tr>
<td>H2 database</td>
<td>Complex</td>
<td>1874 qps</td>
<td>9 (2)</td>
</tr>
<tr>
<td>H2 database</td>
<td>NestedLists</td>
<td>1893 qps</td>
<td>202 (2)</td>
</tr>
<tr>
<td>Cassandra</td>
<td>DynamicEndpointSnitch test</td>
<td>2.907 s</td>
<td>24 (8)</td>
</tr>
</tbody>
</table>

Table 2. Evaluation of FASTTrack and RD2 on two industrial applications. The number variables/objects with races are in brackets.

**Effectiveness.** To measure effectiveness of the race detectors, we compared the following settings for each of the benchmarks: (i) uninstrumented code (ii) using the FASTTrack [3] low-level race detector, and (iii) using RD2.

Then, we compared the runtime overheads and the reported races, summarized in Table 2. For FASTTrack, we show the total number of races and distinct variables on which they occur, and for RD2 we show the total number of commutativity races and the distinct objects on which they occur. In terms of overhead, we found that RD2 imposes a performance penalty similar to FASTTrack, which is expected because even in RD2, RoadRunner instruments reads and writes to all memory locations (plus the ConcurrentHashMap objects for RD2). We note that if we only instrumented the ConcurrentHashMap objects and not the basic memory locations, the overhead of RD2 would be lower.

In terms of precision, most races are highly redundant (meaning that they occur on the same memory locations or on the same concurrent hash map objects). RD2 discovered the following new and interesting commutativity races, which we believe are worth inspecting by the developers:

1. Concurrent accesses to the freedPageSpace map in the MVStore of H2 could lead to incorrect state of the server. The bug is already fixed in the latest (yet unreleased as of November 2013) version of the H2 database, but our tool was able to discover it automatically (in the currently released version).
2. Concurrent accesses to the chunks map in the MVStore of H2 could lead to the same result being computed multiple times, which might be a performance issue if the computation is expensive.
3. New entries to the samples map of the DynamicEndpointSnitch class could be added while its size is concurrently used as a performance hint during node rank recalculation, causing the performance hint to become obsolete.

Overall, we believe that commutativity races are a new and interesting indicator of concurrency problems that can be efficiently discovered with a commutativity race detector.

8. Related Work

In this section, we survey some of the more closely related work on commutativity and concurrency analysis. Most of this work falls in the space of optimistic concurrency control.

**Commutativity.** Commutativity has appeared in various contexts including lower bounds [1], operating system design [2], locking [9], synchronizing abstract data-types [15], concurrency control in transactions [18], thread-level speculation [11], transactional memory [7] and abstract dependence tracking [17].
9. Conclusion and Future Work

In this paper we introduced the concept of a commutativity race and also presented a dynamic commutativity race detector. Our analysis is a novel combination of access point representations with vector clocks. Further, we introduced a logical fragment (ECL) that captures useful commutativity specifications (e.g. maps) and a translation from that fragment to access point representations. A key benefit of ECL is that analysis of objects captured in this fragment requires only a constant number of checks per operation as opposed to linear. We implemented a proof-of-concept of our approach and demonstrated its effectiveness in practice. Conceptually, our work can be seen as a generalization of traditional race detectors.

Interesting items for future work include extending ECL, generalizing other analyzers (e.g. atomicity) and providing optimizing compilers from ECL-like logics to access point representations.

References

[8] Kim, D., And Rinard, M. C. Verification of semantic commutativity conditions and inverse operations on linked data structures. In ACM PLDI’11.
A. Appendix

A.1 Proofs

Proof of Theorem 5.1. Let $e \in \pi$ be the current event that is analyzed. Let $pt'$ be an access point and let $(e_i)_{i=1}^n$ be the set of all events in which $pt'$ was previously touched by some action. The second phase of the algorithm maintains the invariant that $pt'.vc = vc(e_1) \cup \cdots \cup vc(e_n)$ and $pt' \in \text{active}(o)$ iff $n > 0$. Also, from the definitions of $\sqcup$ and $\sqcap$, for any $c \in VC$ we have that $vc(e_1) \cup \cdots \cup vc(e_n) \sqcap e \sqcup e_i$ for each $e_i$.

It is enough to show that the first phase of Algorithm 1 will report a race iff there is a race between $e$ and some other event $e' \leq e$. A race is reported iff there is $pt'$ such that $pt'.vc \sqcup vc(e)$ and $(pt', pt') \in C_o$. Because $pt'.vc = vc(e_1) \cup \cdots \cup vc(e_n)$, this happens iff there exist some $e_i = e'$ such that $vc(e') \sqsubseteq vc(e)$ and $(pt', pt') \in C_o$. The last is equivalent to $(pt', pt') \in C_o$, and $e' \leq e$, but $e' \not\leq e$ which is equivalent to a commutativity race because we work with a representation (Definition 4.5).

\[ \square \]

Proof of Theorem 6.5. Let $\varphi_{m2}^a(x_1\ldots x_k; y_1\ldots y_l) \in \Phi$, and let $a$ and $b$ be some actions of $m_1$ and $m_2$ respectively. It is enough to show that $\varphi_{m2}^a(b, a) \equiv false$ iff there exist two access points $pt_1 \in \eta_p(a), pt_2 \in \eta_p(b)$ such that $(pt_1, pt_2) \in C_o$.

Let $\beta_1$ and $\beta_2$ be the corresponding beta vectors of $a$ and $b$. By the definition of $\beta_1$ and $\beta_2$ we have $\varphi_{m2}^a(b, a) \equiv false$ iff $\varphi_{m2}^b[\beta_1; \beta_2](a, b) \equiv false$. The formula $\varphi_{m2}^b[\beta_1; \beta_2]$ is equivalent to a $LB$ formula, that is, conjunction of literals of the form $x \neq y$. Therefore, $\varphi_{m2}^b[\beta_1; \beta_2](a, b) \equiv false$ if $\varphi_{m2}^b[\beta_1; \beta_2]$ or some of the mentioned conjuncts evaluates to false. By the definition of $C_o$ this is the case iff there are two access points $pt_1 \in \eta_p(a), pt_2 \in \eta_p(b)$ such that $(pt_1, pt_2) \in C_o$.

\[ \square \]

A.2 Translating the dictionary specification

Consider the commutativity specification $\Phi$ of a dictionary object $o$ from Fig. 6. The specification fits the ECL fragment and we will show how to obtain an access point representation from it. The result, however, will not match the one in Fig. 7 which can be seen as an optimized variant. Later, we will consider a few automatic transformations which can achieve this optimization.

We first have to find out the access points for each of the methods. To determine the shape of the $\beta$ components, for each method we need to find out the $LB$ atomic subformulas occurring in the specification. Actually, only $put$ has any:

\[ B(\Phi, put) = \{ v = p, v = nil, p = nil \} \]

Omitting access points that will not appear in the conflict relation, we have:

\[
\begin{align*}
\eta_p(put \ (k, v) / p) &= \{ \text{o.put:} \beta \text{ds, o.put:} \beta \text{1:k} \} \\
\eta_p(get \ (k) / v) &= \{ \text{o.get:} \varnothing \text{1:k} \} \\
\eta_p(size \ (\_ / r)) &= \{ \text{o.size:} \varnothing \text{2:ds} \}
\end{align*}
\]

Where $\beta$ stands for the beta vector of the corresponding action for which $\eta_p$ is being defined.

Now we have to define the conflict relation $C_o$. We will do this only for the $put$ method, the rest being similar. Directly following the definition from Section 6.2 we obtain the conflicts:

\[
\begin{align*}
\text{o.put:} \beta \text{1:u, o.put:} \beta \text{2:1m} & \in C_o \text{ iff } u = v \land (\neg \beta_1(k = v) \lor \neg \beta_2(k = v)) \\
\text{o.put:} \beta \text{1:u, o.get:} \varnothing \text{1:m} & \in C_o \text{ iff } u = v \land \beta_1(k = v) \\
\text{o.put:} \beta \text{1:ds, o.size:} \varnothing \text{2:ds} & \in C_o \text{ iff } \\
\end{align*}
\]

However, the result differs from the access point representation given in Fig. 7. The two, however, are equivalent with respect to Definition 4.5, as can be shown by applying several transformations.

A.3 Simplifying the obtained access point representation

Let us continue the example and examine several rules that can be applied in order to simplify the access point representation of the specification $\Phi$ on Fig. 6.

Consolidation. The domain of $put$’s $\beta$ vector is the set of atoms $B(\Phi, put) = \{ v = p, v = nil, p = nil \}$. However, the two atoms $v = nil$ and $p = nil$ are only used in the $LB$ subformula $(v = nil \land p = nil) \lor (v \neq nil \land p \neq nil)$. To shorten the notation we will use the equivalent $v = nil \iff p = nil$. This formula can replace the two atoms in the domain of $\beta$:

\[
\beta : \{ v = p, v = nil \iff p = nil \} \rightarrow \{ true, false \}
\]

We can consolidate such atoms because their separate valuation is irrelevant.

Dropping. Note that the two subformulas $v = p$ and $v = nil \iff p = nil$ never appear together in a single formula. That is why conflicts with $o.put: \beta 1:u$ do not depend on the truth value of $v = nil \iff p = nil$ and conflicts with $o.put: \beta 2:ds$ do not depend on the truth value of $v = p$. Therefore, in the corresponding access points we can drop the unneeded atoms from the domain of the definition of $\beta$. We obtain four possible $\beta$ vectors:

\[
\begin{align*}
\text{o.put:} \beta \text{1:u} & \rightarrow o:x:u \iff \beta(k = v) \\
\text{o.put:} \beta \text{1:u} & \rightarrow o:x:u \iff \beta(k = v) \\
\text{o.put:} \beta 2:ds & \rightarrow o:noresize \iff \beta(k = nil \iff p = nil) \\
\text{o.put:} \beta 2:ds & \rightarrow o:noresize \iff \beta(k = nil \iff p = nil)
\end{align*}
\]

In general, parts of $\beta$ can be dropped when they do not influence the conflict relation $C_o$.

Cleanup. With the definitions above, the access point representation becomes:

\[
\begin{align*}
\{ o; \beta 1 : u, o; \beta 2 : v \} & \in C_o \text{ iff } u = v \land (\beta_1 = v \lor \beta_2 = v) \\
\{ o; \beta 1 : u, o.get: \varnothing 1 : v \} & \in C_o \text{ iff } u = v \land (\beta_1 = v) \\
\{ o; \beta 1 , o.size() : \varnothing 2 : ds \} & \in C_o \text{ iff } \beta_1 = \text{resize}
\end{align*}
\]

The access point $o:noresize$ is not referenced at all. Such access points are unnecessary and can be safely removed.

Replacement. A non-trivial transformation involves the fact that as far as the conflict relation $C_o$ is concerned, the access points $o:v$ and $o.get: \varnothing 1 : u$ carry the same information, that is:

\[
\{ o; \beta 1 : u, o:v \} \in C_o \leftrightarrow \{ o; \beta 1 : u, o.get: \varnothing 1 : v \} \in C_o
\]

Therefore, $o:v$ can safely be substituted for $o.get: \varnothing 1 : u$, obtaining:

\[
\eta_p(\text{get} \ (k) / v) = \{ \text{o:v} \}
\]

and of course, removing the now redundant access point.

In general, any access points $pt_1$ and $pt_2$ are congruent iff for any third one $pt_3$ we have $(pt_1, pt_3) \in C_o$ iff $(pt_2, pt_3) \in C_o$. Each access point in a congruence class can be replaced with a single representative from that class.