"The following variable defines the 3 intervals of $[0,1]$ on which ψ is linear."

```
interval = {{0, 1/6}, {1/6, 5/6}, {5/6, 1}}
```

Out[48]= $\left\{\left\{0, \frac{1}{6}\right\}, \left\{\frac{1}{6}, \frac{5}{6}\right\}, \left\{\frac{5}{6}, 1\right\}\right\}$

"This is the definition of ψ on each one of the 3 intervals."

In[2]:=
```
psi = { Function[x, 0], Function[x, (6 x - 1) / 4], Function[x, 1]}
```

Out[2]= $\left\{\text{Function}[x, 0], \text{Function}\left[x, \frac{1}{4} (6 x - 1)\right], \text{Function}[x, 1]\right\}$

"These are the three non-symmetric possibilities of $(\t(u,v), \t(v,w), \t(w,u))$."

In[8]:=
```
taus = {{0, 0, 0}, {0, 1, 1}, {1, 1, 1}}
```

Out[8]= {{0, 0, 0}, {0, 1, 1}, {1, 1, 1}}

"The following variable fh will hold $A_L^\psi(u,v,w)$. The variables $psix1, psix2,psix3$ represent $\psi(x_{LP}(u,v)), \psi(x_{LP}(v,w)), \psi(x_{LP}(w,u))$ respectively, and $tau1, tau2,tau3$ represent $\t(u,v),\t(v,w),\t(w,u)$, respectively"

```
fh =
 ((1 - psix1) (1 - psix2) + psix1 (1 - psix2) + (1 - psix1) psix2) (x3 (1 - tau3) + (1 - x3) tau3) +
  ((1 - psix2) (1 - psix3) + psix2 (1 - psix3) + (1 - psix2) psix3) (x1 (1 - tau1) + (1 - x1) tau1) +
  ((1 - psix3) (1 - psix1) + psix3 (1 - psix1) + (1 - psix3) psix1) (x2 (1 - tau2) + (1 - x2) tau2)
```

Out[25]=
```
((1 - psix2) (1 - psix3) + psix2 (1 - psix3) + (1 - psix2) psix3) (tau1 (1 - x1) + (1 - tau1) x1) +
 ((1 - psix1) (1 - psix3) + psix1 (1 - psix3) + (1 - psix1) psix3) (tau2 (1 - x2) + (1 - tau2) x2) +
 ((1 - psix1) (1 - psix2) + psix1 (1 - psix2) + (1 - psix1) psix2) (tau3 (1 - x3) + (1 - tau3) x3)
```

"Similarly we will use the Mathematica variable fx to represent $B_\psi(u,v,w)$."

```
fx = (1 - psix1) (1 - psix2) tau3 + (psix1 (1 - psix2) + (1 - psix1) psix2) (1 - tau3) +
   (1 - psix2) (1 - psix3) tau1 + (psix2 (1 - psix3) + (1 - psix2) psix3) (1 - tau1) +
  (1 - psix3) (1 - psix1) tau2 + (psix3 (1 - psix1) + (1 - psix3) psix1) (1 - tau2)
```

Out[26]=
```
(psix2 (1 - psix3) + (1 - psix2) psix3) (1 - tau1) + (1 - psix2) (1 - psix3) tau1 +
 (psix1 (1 - psix3) + (1 - psix1) psix3) (1 - tau2) + (1 - psix1) (1 - psix3) tau2 +
 (psix1 (1 - psix2) + (1 - psix1) psix2) (1 - tau3) + (1 - psix1) (1 - psix2) tau3
```

"Our goal is to prove that $fx \leq 3fh/2$.
To that end we define a function $ToMaximize$ and we will show that it is nonpositive."

In[27]:=
```
ToMaximize = fx - 3 fh / 2
```

Out[27]=
$$(psix2\,(1-psix3)+(1-psix2)\,psix3)\,(1-tau1)+(1-psix2)\,(1-psix3)\,tau1+$$
$$(psix1\,(1-psix3)+(1-psix1)\,psix3)\,(1-tau2)+(1-psix1)\,(1-psix3)\,tau2+$$
$$(psix1\,(1-psix2)+(1-psix1)\,psix2)\,(1-tau3)+(1-psix1)\,(1-psix2)\,tau3-$$
$$\frac{3}{2}\,(((1-psix2)\,(1-psix3)+psix2\,(1-psix3)+(1-psix2)\,psix3)\,(tau1\,(1-x1)+(1-tau1)\,x1)+$$
$$((1-psix1)\,(1-psix3)+psix1\,(1-psix3)+(1-psix1)\,psix3)\,(tau2\,(1-x2)+(1-tau2)\,x2)+$$
$$((1-psix1)\,(1-psix2)+psix1\,(1-psix2)+(1-psix1)\,psix2)\,(tau3\,(1-x3)+(1-tau3)\,x3))$$

```
"The following variable $ToMaximizeijk$ is a function that takes $4$ indices as input,
  and returns the function $ToMaximize$ with $psix1,psix2,psix3$
replaced by $\psi_i(x_{uv}), \psi_j(x_{vw}),
  \psi_k(x_{wu})$ where $\psi_1,\psi_2,\psi_3$ correspond to the definition of $\psi$ on the
   three
intervals on which it is linear, respectively.
  The variables $tau1,tau2,tau3$ are substituted by the possibility pointed to by the
  argument $tauindex$."

ToMaximizeijk = Function[{i, j, k, tauindex},
   ToMaximize /. {psix1 → psi[[i]][x1], psix2 → psi[[j]][x2], psix3 → psi[[k]][x3],
     tau1 -> taus[[tauindex]][[1]],
     tau2 -> taus[[tauindex]][[2]], tau3 -> taus[[tauindex]][[3]]}]

"The following loop tries all posibilities for $i,j,k, tauindex\in \{1,2,3\}$,
  and for each one computes the maximum of $ToMaximizeijk(i,j,k,tauindex)$
in the polytope defined by the corresponding constraints.
  One checks that the maximal value is always $\leq 0$."
```

Out[28]=
```
Function[{i, j, k, tauindex},
 ToMaximize /. {psix1 → psi⟦i⟧[x1], psix2 → psi⟦j⟧[x2], psix3 → psi⟦k⟧[x3],
   tau1 → taus⟦tauindex⟧⟦1⟧, tau2 → taus⟦tauindex⟧⟦2⟧, tau3 → taus⟦tauindex⟧⟦3⟧}]
```

```
For[i = 1, i ≤ 3, i++,
 For[j = 1, j ≤ 3, j++,
  For[k = 1, k ≤ 3, k++,
   For[tauindex = 1, tauindex ≤ 3, tauindex++,
    Print[
     "-------------------------------------------------------------------------------------------
       ---------------"];
    Print["i,j,k,tau = ", i, ",", j, ",", k, ",", taus[[tauindex]]];
    Print["Function=", Simplify[ToMaximizeijk[i, j, k, tauindex]]];
    polytope =
     {x1 ≤ interval[[i]][[2]], x1 ≥ interval[[i]][[1]],
      x2 ≤ interval[[j]][[2]], x2 ≥ interval[[j]][[1]],
      x3 ≤ interval[[k]][[2]], x3 ≥ interval[[k]][[1]],
      x1 ≤ x2 + x3,  x2 ≤ x3 + x1,  x3 ≤ x1 + x2};
    Print["Polytope=", polytope];
    output = Maximize[{ToMaximizeijk[i, j, k, tauindex], polytope}, {x1, x2, x3}];
    Print["Maximum=", output[[1]], " obtained at: ", output[[2]]]]]]]
```

```
-------------------------------------------------------------------------------------------
  -----

i,j,k,tau = 1,1,1,{0, 0, 0}

Function=-3/2 (x1 + x2 + x3)

Polytope={x1 ≤ 1/6, x1 ≥ 0, x2 ≤ 1/6, x2 ≥ 0, x3 ≤ 1/6, x3 ≥ 0, x1 ≤ x2 + x3, x2 ≤ x1 + x3, x3 ≤ x1 + x2}

Maximum=0 obtained at: {x1 → 0, x2 → 0, x3 → 0}

-------------------------------------------------------------------------------------------
  -----
```

i,j,k,tau = 1,1,1,{0, 1, 1}

Function=$\frac{1}{2}$ (−2 − 3 x1 + 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to 0,\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 1,1,1,{1, 1, 1}

Function=$\frac{3}{2}$ (−1 + x1 + x2 + x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{3}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 1,1,2,{0, 0, 0}

Function=$\frac{1}{2}$ (−1 − 3 x1 − 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to 0,\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 1,1,2,{0, 1, 1}

Function=$\frac{1}{2}$ (−2 − 3 x1 + 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to 0,\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 1,1,2,{1, 1, 1}

Function=$\frac{1}{2}$ (−2 + 3 x1 + 3 x2 − 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{3}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 1,1,3,{0, 0, 0}

Function=$2-\frac{3}{2}(x1+x2+x3)$

Polytope=$\left\{x1\le\frac{1}{6},\ x1\ge 0,\ x2\le\frac{1}{6},\ x2\ge 0,\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximize::infeas : There are no values of {x1, x2, x3} for which the constraints

$x1\le\frac{1}{6}\ \&\&\ x1\ge 0\ \&\&\ x2\le\frac{1}{6}\ \&\&\ x2\ge 0\ \&\&\ x3\le 1\ \&\&\ x3\ge\frac{5}{6}\ \&\&\ x1\le x2+x3\ \&\&\ x2\le x1+x3\ \&\&\ x3\le x1+x2$

are satisfied and the objective function $2-\frac{3}{2}(x1+x2+x3)$ is real-valued. >>

Maximum=$-\infty$ obtained at: {x1 → Indeterminate, x2 → Indeterminate, x3 → Indeterminate}

---

i,j,k,tau = 1,1,3,{0, 1, 1}

Function=$\frac{1}{2}(-2-3\,x1+3\,x2+3\,x3)$

Polytope=$\left\{x1\le\frac{1}{6},\ x1\ge 0,\ x2\le\frac{1}{6},\ x2\ge 0,\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximize::infeas : There are no values of {x1, x2, x3} for which the constraints

$x1\le\frac{1}{6}\ \&\&\ x1\ge 0\ \&\&\ x2\le\frac{1}{6}\ \&\&\ x2\ge 0\ \&\&\ x3\le 1\ \&\&\ x3\ge\frac{5}{6}\ \&\&\ x1\le x2+x3\ \&\&\ x2\le x1+x3\ \&\&\ x3\le x1+x2$

are satisfied and the objective function $2-\frac{3}{2}(2+x1-x2-x3)$ is real-valued. >>

Maximum=$-\infty$ obtained at: {x1 → Indeterminate, x2 → Indeterminate, x3 → Indeterminate}

---

i,j,k,tau = 1,1,3,{1, 1, 1}

Function=$\frac{1}{2}(-7+3\,x1+3\,x2+3\,x3)$

Polytope=$\left\{x1\le\frac{1}{6},\ x1\ge 0,\ x2\le\frac{1}{6},\ x2\ge 0,\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximize::infeas : There are no values of {x1, x2, x3} for which the constraints

$x1\le\frac{1}{6}\ \&\&\ x1\ge 0\ \&\&\ x2\le\frac{1}{6}\ \&\&\ x2\ge 0\ \&\&\ x3\le 1\ \&\&\ x3\ge\frac{5}{6}\ \&\&\ x1\le x2+x3\ \&\&\ x2\le x1+x3\ \&\&\ x3\le x1+x2$

are satisfied and the objective function $1-\frac{3}{2}(3-x1-x2-x3)$ is real-valued. >>

General::stop :
Further output of Maximize::infeas will be suppressed during this calculation. >>

Maximum=$-\infty$ obtained at: {x1 → Indeterminate, x2 → Indeterminate, x3 → Indeterminate}

---

i,j,k,tau = 1,2,1,{0, 0, 0}

Function=$\frac{1}{2}$ (−1 − 3 x1 + 3 x2 − 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{1}{6},\ x3 \to 0\right\}$

---

i,j,k,tau = 1,2,1,{0, 1, 1}

Function=$\frac{1}{2}$ (−2 − 3 x1 + 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to 0,\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 1,2,1,{1, 1, 1}

Function=$\frac{1}{2}$ (−2 + 3 x1 − 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{3}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 1,2,2,{0, 0, 0}

Function=$\frac{9}{32}$ (−4 (−1 + 2 x2) (−1 + 2 x3) + x1 (−5 − 2 x3 + 2 x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to 0,\ x2 \to \frac{1}{2},\ x3 \to \frac{1}{2}\right\}$

---

i,j,k,tau = 1,2,2,{0, 1, 1}

Function=$\frac{9}{32}$ (−4 (−1 + 2 x2) (−1 + 2 x3) + x1 (−5 − 2 x3 + 2 x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to 0,\ x2 \to \frac{1}{2},\ x3 \to \frac{1}{2}\right\}$

---

i,j,k,tau = 1,2,2,{1, 1, 1}

Function=$\frac{1}{32}$ (−11 − 78 x3 + 6 x2 (−13 + 30 x3) − 9 x1 (−5 − 2 x3 + 2 x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to \frac{5}{64},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 1,2,3,{0, 0, 0}

Function=$\frac{1}{8}$ (3 x1 (−5 + 6 x2) − 4 (−4 + 3 x2 + 3 x3))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{5}{16}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{2}{3},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 1,2,3,{0, 1, 1}

Function=$\frac{1}{8}$ (3 x1 (−5 + 6 x2) − 4 (1 + 3 x2 − 3 x3))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{5}{6},\ x3 \to 1\right\}$

---

i,j,k,tau = 1,2,3,{1, 1, 1}

Function=$\frac{1}{8}$ (−29 + 18 x2 − 3 x1 (−5 + 6 x2) + 12 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{5}{6},\ x3 \to 1\right\}$

---

i,j,k,tau = 1,3,1,{0, 0, 0}

Function=$2 - \frac{3}{2}$ (x1 + x2 + x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\infty$ obtained at: {x1 → Indeterminate, x2 → Indeterminate, x3 → Indeterminate}

---

i,j,k,tau = 1,3,1,{0, 1, 1}

Function=$\frac{1}{2}$ (−2 − 3 x1 + 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\infty$ obtained at: {x1 → Indeterminate, x2 → Indeterminate, x3 → Indeterminate}

---

i,j,k,tau = 1,3,1,{1, 1, 1}

Function=$\frac{1}{2}$ (−7 + 3 x1 + 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\infty$ obtained at: {x1 → Indeterminate, x2 → Indeterminate, x3 → Indeterminate}

---

i,j,k,tau = 1,3,2,{0, 0, 0}

Function=$\frac{1}{8}$ (−4 (−4 + 3 x2 + 3 x3) + 3 x1 (−5 + 6 x3))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{5}{16}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{2}{3}\right\}$

---

i,j,k,tau = 1,3,2,{0, 1, 1}

Function=$\frac{1}{8}$ (4 (−1 + 3 x2 − 3 x3) + 3 x1 (−5 + 6 x3))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to 1,\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 1,3,2,{1, 1, 1}

Function=$\frac{1}{8}$ (−29 + 12 x2 + 18 x3 − 3 x1 (−5 + 6 x3))

Polytope=$\left\{x1 \le \frac{1}{6},\ x1 \ge 0,\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to 1,\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 1,3,3,{0, 0, 0}

Function=$2-\frac{3\,(x2+x3)}{2}$

Polytope=$\left\{x1\le\frac{1}{6},\ x1\ge 0,\ x2\le 1,\ x2\ge\frac{5}{6},\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1\to 0,\ x2\to\frac{5}{6},\ x3\to\frac{5}{6}\right\}$

---

i,j,k,tau = 1,3,3,{0, 1, 1}

Function=$\frac{3}{2}\,(-2+x2+x3)$

Polytope=$\left\{x1\le\frac{1}{6},\ x1\ge 0,\ x2\le 1,\ x2\ge\frac{5}{6},\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=0 obtained at: $\{x1\to 0,\ x2\to 1,\ x3\to 1\}$

---

i,j,k,tau = 1,3,3,{1, 1, 1}

Function=$\frac{3}{2}\,(-2+x2+x3)$

Polytope=$\left\{x1\le\frac{1}{6},\ x1\ge 0,\ x2\le 1,\ x2\ge\frac{5}{6},\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=0 obtained at: $\{x1\to 0,\ x2\to 1,\ x3\to 1\}$

---

i,j,k,tau = 2,1,1,{0, 0, 0}

Function=$\frac{1}{2}\,(-1+3\,x1-3\,x2-3\,x3)$

Polytope=$\left\{x1\le\frac{5}{6},\ x1\ge\frac{1}{6},\ x2\le\frac{1}{6},\ x2\ge 0,\ x3\le\frac{1}{6},\ x3\ge 0,\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1\to\frac{1}{6},\ x2\to 0,\ x3\to\frac{1}{6}\right\}$

---

i,j,k,tau = 2,1,1,{0, 1, 1}

Function=$\frac{1}{2}\,(-1-9\,x1+3\,x2+3\,x3)$

Polytope=$\left\{x1\le\frac{5}{6},\ x1\ge\frac{1}{6},\ x2\le\frac{1}{6},\ x2\ge 0,\ x3\le\frac{1}{6},\ x3\ge 0,\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=$-\frac{3}{4}$ obtained at: $\left\{x1\to\frac{1}{6},\ x2\to\frac{1}{6},\ x3\to\frac{1}{6}\right\}$

---

i,j,k,tau = 2,1,1,{1, 1, 1}

Function=$\frac{1}{2}$ (−2 − 3 x1 + 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{3}{4}$ obtained at: $\left\{x1 \to \frac{1}{6},\ x2 \to \frac{1}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 2,1,2,{0, 0, 0}

Function=$\frac{9}{32}$ (−4 + 8 x3 − x2 (5 + 2 x3) + 2 x1 (4 − 8 x3 + x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{1}{2},\ x2 \to 0,\ x3 \to \frac{1}{2}\right\}$

---

i,j,k,tau = 2,1,2,{0, 1, 1}

Function=$\frac{1}{32}$ (−11 + 18 x3 + 9 x2 (5 + 2 x3) − 6 x1 (29 − 30 x3 + 3 x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{5}{16}$ obtained at: $\left\{x1 \to \frac{2}{3},\ x2 \to \frac{1}{6},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 2,1,2,{1, 1, 1}

Function=$\frac{1}{32}$ (−11 − 78 x3 + 9 x2 (5 + 2 x3) − 6 x1 (13 − 30 x3 + 3 x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{64},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 2,1,3,{0, 0, 0}

Function=$\frac{1}{8}$ (16 − 15 x2 + 6 x1 (−2 + 3 x2) − 12 x3)

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{5}{16}$ obtained at: $\left\{x1 \to \frac{2}{3}, x2 \to \frac{1}{6}, x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 2,1,3,{0, 1, 1}

Function=$-\frac{3}{8}(3 - 5\,x2 + x1\,(2 + 6\,x2) - 4\,x3)$

Polytope=$\left\{x1 \le \frac{5}{6}, x1 \ge \frac{1}{6}, x2 \le \frac{1}{6}, x2 \ge 0, x3 \le 1, x3 \ge \frac{5}{6}, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{5}{6}, x2 \to \frac{1}{6}, x3 \to 1\right\}$

---

i,j,k,tau = 2,1,3,{1, 1, 1}

Function=$\frac{1}{8}(-29 - 18\,x1\,(-1 + x2) + 15\,x2 + 12\,x3)$

Polytope=$\left\{x1 \le \frac{5}{6}, x1 \ge \frac{1}{6}, x2 \le \frac{1}{6}, x2 \ge 0, x3 \le 1, x3 \ge \frac{5}{6}, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{5}{6}, x2 \to \frac{1}{6}, x3 \to 1\right\}$

---

i,j,k,tau = 2,2,1,{0, 0, 0}

Function=$\frac{9}{32}(-4 - 2\,x2\,(-4 + x3) - 5\,x3 + 2\,x1\,(4 - x3 + x2\,(-8 + 6\,x3)))$

Polytope=$\left\{x1 \le \frac{5}{6}, x1 \ge \frac{1}{6}, x2 \le \frac{5}{6}, x2 \ge \frac{1}{6}, x3 \le \frac{1}{6}, x3 \ge 0, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{1}{2}, x2 \to \frac{1}{2}, x3 \to 0\right\}$

---

i,j,k,tau = 2,2,1,{0, 1, 1}

Function=$\frac{1}{32}(-11 + 45\,x3 + 18\,x2\,(1 + x3) - 6\,x1\,(29 - 3\,x3 + 6\,x2\,(-5 + 3\,x3)))$

Polytope=$\left\{x1 \le \frac{5}{6}, x1 \ge \frac{1}{6}, x2 \le \frac{5}{6}, x2 \ge \frac{1}{6}, x3 \le \frac{1}{6}, x3 \ge 0, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=$-\frac{5}{16}$ obtained at: $\left\{x1 \to \frac{2}{3}, x2 \to \frac{5}{6}, x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 2,2,1,{1, 1, 1}

Function=$\frac{1}{32}(-11 + 45\,x3 + 6\,x2\,(-13 + 3\,x3) - 6\,x1\,(13 - 3\,x3 + 6\,x2\,(-5 + 3\,x3)))$

$$\text{Polytope}=\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2+x3,\ x2 \le x1+x3,\ x3 \le x1+x2\right\}$$

$$\text{Maximum}=-\frac{1}{2} \text{ obtained at: } \left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{64}\right\}$$

$$i,j,k,\text{tau} = 2,2,2,\{0,\ 0,\ 0\}$$

$$\text{Function}=\frac{3}{32}\left(-20+x2\left(33-60\,x3\right)+33\,x3+3\,x1\left(11-20\,x3+4\,x2\left(-5+9\,x3\right)\right)\right)$$

$$\text{Polytope}=\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2+x3,\ x2 \le x1+x3,\ x3 \le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$$

$$i,j,k,\text{tau} = 2,2,2,\{0,\ 1,\ 1\}$$

$$\text{Function}=\frac{1}{32}\left(-10+x2\left(39-108\,x3\right)+39\,x3-3\,x1\left(67-60\,x3+12\,x2\left(-5+3\,x3\right)\right)\right)$$

$$\text{Polytope}=\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2+x3,\ x2 \le x1+x3,\ x3 \le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$$

$$i,j,k,\text{tau} = 2,2,2,\{1,\ 1,\ 1\}$$

$$\text{Function}=-\frac{3}{32}\left(-5+x2\left(37-72\,x3\right)+37\,x3+x1\left(37-72\,x3+36\,x2\left(-2+3\,x3\right)\right)\right)$$

$$\text{Polytope}=\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2+x3,\ x2 \le x1+x3,\ x3 \le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$$

$$i,j,k,\text{tau} = 2,2,3,\{0,\ 0,\ 0\}$$

$$\text{Function}=\frac{3}{32}\left(20-15\,x3-6\,x2\left(2+x3\right)+6\,x1\left(-2+\left(-1+6\,x2\right)x3\right)\right)$$

$$\text{Polytope}=\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2+x3,\ x2 \le x1+x3,\ x3 \le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{59}{64}\right\}$$

$$i,j,k,\text{tau} = 2,2,3,\{0,\ 1,\ 1\}$$

Function=$-\frac{3}{32}(5+x2(22-6x3)-15x3+x1(22-6x3+12x2(-5+3x3)))$

Polytope=$\left\{x1\le\frac{5}{6},\ x1\ge\frac{1}{6},\ x2\le\frac{5}{6},\ x2\ge\frac{1}{6},\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=0 obtained at: $\left\{x1\to\frac{1}{2},\ x2\to\frac{1}{2},\ x3\to 1\right\}$

---

i,j,k,tau = 2,2,3,{1, 1, 1}

Function=$\frac{1}{32}(18x2(3+x3)+5(-23+9x3)-18x1(-3-x3+x2(-2+6x3)))$

Polytope=$\left\{x1\le\frac{5}{6},\ x1\ge\frac{1}{6},\ x2\le\frac{5}{6},\ x2\ge\frac{1}{6},\ x3\le 1,\ x3\ge\frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=0 obtained at: $\left\{x1\to\frac{5}{6},\ x2\to\frac{5}{6},\ x3\to\frac{59}{64}\right\}$

---

i,j,k,tau = 2,3,1,{0, 0, 0}

Function=$\frac{1}{8}(16-12x2-15x3+6x1(-2+3x3))$

Polytope=$\left\{x1\le\frac{5}{6},\ x1\ge\frac{1}{6},\ x2\le 1,\ x2\ge\frac{5}{6},\ x3\le\frac{1}{6},\ x3\ge 0,\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=$-\frac{5}{16}$ obtained at: $\left\{x1\to\frac{2}{3},\ x2\to\frac{5}{6},\ x3\to\frac{1}{6}\right\}$

---

i,j,k,tau = 2,3,1,{0, 1, 1}

Function=$\frac{3}{8}(-3+4x2+5x3-2x1(1+3x3))$

Polytope=$\left\{x1\le\frac{5}{6},\ x1\ge\frac{1}{6},\ x2\le 1,\ x2\ge\frac{5}{6},\ x3\le\frac{1}{6},\ x3\ge 0,\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1\to\frac{5}{6},\ x2\to 1,\ x3\to\frac{1}{6}\right\}$

---

i,j,k,tau = 2,3,1,{1, 1, 1}

Function=$\frac{1}{8}(-29+12x2-18x1(-1+x3)+15x3)$

Polytope=$\left\{x1\le\frac{5}{6},\ x1\ge\frac{1}{6},\ x2\le 1,\ x2\ge\frac{5}{6},\ x3\le\frac{1}{6},\ x3\ge 0,\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1\to\frac{5}{6},\ x2\to 1,\ x3\to\frac{1}{6}\right\}$

---

i,j,k,tau = 2,3,2,{0, 0, 0}

Function=$\frac{3}{32}$ (20 − 12 x3 − 3 x2 (5 + 2 x3) + 6 x1 (−2 + x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{59}{64},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 2,3,2,{0, 1, 1}

Function=$-\frac{3}{32}$ (5 + 22 x3 − 3 x2 (5 + 2 x3) + x1 (22 − 60 x3 + 6 x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{59}{64},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 2,3,2,{1, 1, 1}

Function=$\frac{1}{32}$ (−115 + 54 x3 + 9 x2 (5 + 2 x3) − 18 x1 (−3 − 2 x3 + x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{55}{64},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 2,3,3,{0, 0, 0}

Function=$\frac{1}{8}$ (−5 + 6 x1) (−4 + 3 x2 + 3 x3)

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{59}{64},\ x3 \to \frac{59}{64}\right\}$

---

i,j,k,tau = 2,3,3,{0, 1, 1}

Function=$-\frac{3}{8}$ (−5 + 6 x1) (−2 + x2 + x3)

Polytope=$\left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{7}{8},\ x3 \to \frac{59}{64}\right\}$

$$i,j,k,\text{tau} = 2,3,3,\{1, 1, 1\}$$

$$\text{Function} = -\frac{3}{8}\,(-5 + 6\,x1)\,(-2 + x2 + x3)$$

$$\text{Polytope} = \left\{x1 \le \frac{5}{6},\ x1 \ge \frac{1}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$$

$$\text{Maximum} = 0 \text{ obtained at: } \left\{x1 \to \frac{5}{6},\ x2 \to \frac{7}{8},\ x3 \to \frac{59}{64}\right\}$$

$$i,j,k,\text{tau} = 3,1,1,\{0, 0, 0\}$$

$$\text{Function} = 2 - \frac{3}{2}\,(x1 + x2 + x3)$$

$$\text{Polytope} = \left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$$

$$\text{Maximum} = -\infty \text{ obtained at: } \{x1 \to \text{Indeterminate},\ x2 \to \text{Indeterminate},\ x3 \to \text{Indeterminate}\}$$

$$i,j,k,\text{tau} = 3,1,1,\{0, 1, 1\}$$

$$\text{Function} = -\frac{3}{2}\,(2 + x1 - x2 - x3)$$

$$\text{Polytope} = \left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$$

$$\text{Maximum} = -\infty \text{ obtained at: } \{x1 \to \text{Indeterminate},\ x2 \to \text{Indeterminate},\ x3 \to \text{Indeterminate}\}$$

$$i,j,k,\text{tau} = 3,1,1,\{1, 1, 1\}$$

$$\text{Function} = \frac{1}{2}\,(-7 + 3\,x1 + 3\,x2 + 3\,x3)$$

$$\text{Polytope} = \left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$$

$$\text{Maximum} = -\infty \text{ obtained at: } \{x1 \to \text{Indeterminate},\ x2 \to \text{Indeterminate},\ x3 \to \text{Indeterminate}\}$$

$$i,j,k,\text{tau} = 3,1,2,\{0, 0, 0\}$$

$$\text{Function} = \frac{1}{8}\,(16 - 12\,x1 - 12\,x3 + 3\,x2\,(-5 + 6\,x3))$$

$$\text{Polytope} = \left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$$

$$\text{Maximum} = -\frac{5}{16} \text{ obtained at: } \left\{x1 \to \frac{5}{6},\ x2 \to \frac{1}{6},\ x3 \to \frac{2}{3}\right\}$$

---

i,j,k,tau = 3,1,2,{0, 1, 1}

Function=$\frac{1}{8}$ (−29 − 12 x1 + 42 x3 − 3 x2 (−5 + 6 x3))

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{7}{128},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,1,2,{1, 1, 1}

Function=$\frac{1}{8}$ (−29 + 12 x1 + 18 x3 − 3 x2 (−5 + 6 x3))

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to 1,\ x2 \to \frac{1}{6},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,1,3,{0, 0, 0}

Function=$2 - \frac{3\,(x1 + x3)}{2}$

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to 0,\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,1,3,{0, 1, 1}

Function=$\frac{1}{2}$ (−1 − 3 x1 + 3 x3)

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{1}{6},\ x3 \to 1\right\}$

---

i,j,k,tau = 3,1,3,{1, 1, 1}

Function=$\frac{3}{2}$ (−2 + x1 + x3)

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{1}{6},\ x2 \ge 0,\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: {x1 → 1, x2 → 0, x3 → 1}

---

i,j,k,tau = 3,2,1,{0, 0, 0}

Function=$\frac{1}{8}$ (16 − 12 x1 − 15 x3 + 6 x2 (−2 + 3 x3))

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{5}{16}$ obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{2}{3},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 3,2,1,{0, 1, 1}

Function=$\frac{1}{8}$ (−29 − 12 x1 + 15 x3 − 6 x2 (−7 + 3 x3))

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{64}\right\}$

---

i,j,k,tau = 3,2,1,{1, 1, 1}

Function=$\frac{1}{8}$ (−29 + 12 x1 − 18 x2 (−1 + x3) + 15 x3)

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{1}{6},\ x3 \ge 0,\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to 1,\ x2 \to \frac{5}{6},\ x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 3,2,2,{0, 0, 0}

Function=$\frac{3}{32}$ (−4 (−5 + 3 x2 + 3 x3) + 3 x1 (−5 − 2 x3 + 2 x2 (−1 + 6 x3)))

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{59}{64},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,2,2,{0, 1, 1}

Function=$\frac{1}{32}$ (9 x1 (−5 − 2 x3 + 2 x2 (−1 + 6 x3)) − 4 (35 − 51 x3 + x2 (−51 + 72 x3)))

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le \frac{5}{6},\ x2 \ge \frac{1}{6},\ x3 \le \frac{5}{6},\ x3 \ge \frac{1}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{59}{64},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,2,2,{1, 1, 1}

$$\text{Function}=\frac{1}{32}\left(-115+54\,x3+18\,x2\,(3+2\,x3)-9\,x1\,(-5-2\,x3+2\,x2\,(-1+6\,x3))\right)$$

$$\text{Polytope}=\left\{x1\le 1,\ x1\ge \frac{5}{6},\ x2\le \frac{5}{6},\ x2\ge \frac{1}{6},\ x3\le \frac{5}{6},\ x3\ge \frac{1}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1\to \frac{59}{64},\ x2\to \frac{5}{6},\ x3\to \frac{5}{6}\right\}$$

---

i,j,k,tau = 3,2,3,{0, 0, 0}

$$\text{Function}=\frac{1}{8}\left(-5+6\,x2\right)\left(-4+3\,x1+3\,x3\right)$$

$$\text{Polytope}=\left\{x1\le 1,\ x1\ge \frac{5}{6},\ x2\le \frac{5}{6},\ x2\ge \frac{1}{6},\ x3\le 1,\ x3\ge \frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1\to \frac{59}{64},\ x2\to \frac{5}{6},\ x3\to \frac{59}{64}\right\}$$

---

i,j,k,tau = 3,2,3,{0, 1, 1}

$$\text{Function}=\frac{1}{8}\left(-5+6\,x2\right)\left(1+3\,x1-3\,x3\right)$$

$$\text{Polytope}=\left\{x1\le 1,\ x1\ge \frac{5}{6},\ x2\le \frac{5}{6},\ x2\ge \frac{1}{6},\ x3\le 1,\ x3\ge \frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1\to \frac{59}{64},\ x2\to \frac{5}{6},\ x3\to \frac{59}{64}\right\}$$

---

i,j,k,tau = 3,2,3,{1, 1, 1}

$$\text{Function}=-\frac{3}{8}\left(-5+6\,x2\right)\left(-2+x1+x3\right)$$

$$\text{Polytope}=\left\{x1\le 1,\ x1\ge \frac{5}{6},\ x2\le \frac{5}{6},\ x2\ge \frac{1}{6},\ x3\le 1,\ x3\ge \frac{5}{6},\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$$

$$\text{Maximum}=0 \text{ obtained at: } \left\{x1\to 1,\ x2\to \frac{1}{4},\ x3\to 1\right\}$$

---

i,j,k,tau = 3,3,1,{0, 0, 0}

$$\text{Function}=2-\frac{3\,(x1+x2)}{2}$$

$$\text{Polytope}=\left\{x1\le 1,\ x1\ge \frac{5}{6},\ x2\le 1,\ x2\ge \frac{5}{6},\ x3\le \frac{1}{6},\ x3\ge 0,\ x1\le x2+x3,\ x2\le x1+x3,\ x3\le x1+x2\right\}$$

Maximum=$-\frac{1}{2}$ obtained at: $\left\{x1 \to \frac{5}{6}, x2 \to \frac{5}{6}, x3 \to 0\right\}$

---

i,j,k,tau = 3,3,1,{0, 1, 1}

Function=$\frac{1}{2}(-1 - 3\,x1 + 3\,x2)$

Polytope=$\left\{x1 \le 1, x1 \ge \frac{5}{6}, x2 \le 1, x2 \ge \frac{5}{6}, x3 \le \frac{1}{6}, x3 \ge 0, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=$-\frac{1}{4}$ obtained at: $\left\{x1 \to \frac{5}{6}, x2 \to 1, x3 \to \frac{1}{6}\right\}$

---

i,j,k,tau = 3,3,1,{1, 1, 1}

Function=$\frac{3}{2}(-2 + x1 + x2)$

Polytope=$\left\{x1 \le 1, x1 \ge \frac{5}{6}, x2 \le 1, x2 \ge \frac{5}{6}, x3 \le \frac{1}{6}, x3 \ge 0, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=0 obtained at: {x1 → 1, x2 → 1, x3 → 0}

---

i,j,k,tau = 3,3,2,{0, 0, 0}

Function=$\frac{1}{8}(-4 + 3\,x1 + 3\,x2)(-5 + 6\,x3)$

Polytope=$\left\{x1 \le 1, x1 \ge \frac{5}{6}, x2 \le 1, x2 \ge \frac{5}{6}, x3 \le \frac{5}{6}, x3 \ge \frac{1}{6}, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{59}{64}, x2 \to \frac{59}{64}, x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,3,2,{0, 1, 1}

Function=$\frac{1}{8}(1 + 3\,x1 - 3\,x2)(-5 + 6\,x3)$

Polytope=$\left\{x1 \le 1, x1 \ge \frac{5}{6}, x2 \le 1, x2 \ge \frac{5}{6}, x3 \le \frac{5}{6}, x3 \ge \frac{1}{6}, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{59}{64}, x2 \to \frac{59}{64}, x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,3,2,{1, 1, 1}

Function=$-\frac{3}{8}(-2 + x1 + x2)(-5 + 6\,x3)$

Polytope=$\left\{x1 \le 1, x1 \ge \frac{5}{6}, x2 \le 1, x2 \ge \frac{5}{6}, x3 \le \frac{5}{6}, x3 \ge \frac{1}{6}, x1 \le x2 + x3, x2 \le x1 + x3, x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to 1,\ x2 \to 1,\ x3 \to \frac{1}{4}\right\}$

---

i,j,k,tau = 3,3,3,{0, 0, 0}

Function=0

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,3,3,{0, 1, 1}

Function=0

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$

---

i,j,k,tau = 3,3,3,{1, 1, 1}

Function=0

Polytope=$\left\{x1 \le 1,\ x1 \ge \frac{5}{6},\ x2 \le 1,\ x2 \ge \frac{5}{6},\ x3 \le 1,\ x3 \ge \frac{5}{6},\ x1 \le x2 + x3,\ x2 \le x1 + x3,\ x3 \le x1 + x2\right\}$

Maximum=0 obtained at: $\left\{x1 \to \frac{5}{6},\ x2 \to \frac{5}{6},\ x3 \to \frac{5}{6}\right\}$