CPSC 431 : Introduction to Computer Music
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Date: October 16, 2006

Assignment 1 Solution

Problem 1

Let $\omega = 2\pi f$, the sum of odd overtones (or even harmonics) is

$$signal(t) \quad = \quad \sum_{n=1}^{\infty} \frac{1}{2n} \sin((2n)\omega t)$$

Problem 2

We need the overtones to be the multiples of 20Hz, and since the upper bound of human hearing is 20kHz, there could be up to 1000 overtones.

Problem 3

In this case, two speakers would produce the same wave with opposite phase, so they would cancel each other at the position of the listener.

Problem 4

Suppose the original wave is $s(t)$, and at the position of the listener, one wave $s(t)$ traveled 20*ft* and the other wave $-s(t)$ traveled 10*ft*, the sum of them is $s(t + \frac{20}{v}) - s(t + \frac{10}{v})$, where v is the speed of sound.

For them to cancel out, we need the sum to be 0. The frequency of such sine wave component in s will be f that satisfies the following equation for all t

$$\sin(2\pi f(t + \frac{20}{v})) - \sin(2\pi f(t + \frac{10}{v})) = 0$$

which, by the trigonometric identity $\sin(A) - \sin(B) = 2\sin(\frac{A-B}{2})\cos(\frac{A+B}{2})$, becomes

$$2\sin(2\pi f\frac{5}{v})\cos(2\pi f(t + \frac{15}{v})) = 0$$

or, equivalently when $\frac{10}{v} f$ is an integer. Given that $v \approx 1100 ft/s$, f has to be the multiples of $\frac{v}{10} = 110Hz$.

Problem 5

The sound will take $\frac{300}{v}$ seconds to reach the middle of the hall after being reflected by the wall, where v is the speed of sound. Compared to the sound wave directly from the stage, the time delay is $\frac{300-100}{v} = \frac{200}{v} =\approx 0.1818s$. The phase shift for one cycle is the fractional part of $\frac{200f}{v}$ for frequency f. For frequencies 100Hz, 1000Hz, and 5000Hz, the respective phase shifts are 0.1818, 0.8182, and 0.0909 cycle, or 1.142, 5.141, and 0.571 in RAD.

Sitting at the very rear of the room would give me the least amount of phase distortion, because the wave directly from the stage and the wave reflected from the wall would travel almost the same distance, so the phase distortion is negligible.

Problem 6

The distance between my ears is about 8 inches, or $d = 0.67$ feet. The time shift would be the difference beteen the same sound wave travels from the source to both ears, which is at most $0.67/1100 = 0.000609$ second. But the phrase shift would be frequency dependent, so it isn't an important cue for localizing sound.

Problem 7

$$\begin{aligned}
\frac{vol_{100}}{vol_1} &= (\frac{1}{100})^2 \\
10\log(\frac{vol_{100}}{vol_1}) &= 10\log((\frac{1}{100})^2) \\
10\log(vol_{100}) - 10\log(vol_1) &= -40 \\
10\log(vol_{100}) &= 10\log(vol_1) - 40 = 100 - 40 = 60 \quad dB
\end{aligned}$$

Problem 8

$$10\log(\frac{2v}{v}) = 10\log 2 = 3.01 \quad dB$$

Problem 9

http://cnx.org/content/m13687/latest/, section *Temporal Aliasing*.

Problem 10

Vonage is a VOIP company, and less network bandwidth used to transfer voice means less cost and greater capacity as a service provider. To digitize voice message using just 2 kHz bandwidth means a sampling rate of 4 kHz, which is 4000 sampling point per second. If each point is represented by a 2 byte integer, it only consumes $4000 * 2 * 8 =$

$64kbps$ network bandwidth, which is a big save compared to $640kbps$ if all 20 kHz is digitized.

Problem 11

The Fourier transform of $x(t) = sin(\omega t)$ is

$$\begin{aligned} X(f) &= \int_{-\infty}^{+\infty} \sin(\omega t) e^{-i2\pi f t} dt \\ &= \int_{-\infty}^{+\infty} \frac{e^{i\omega t} - e^{-i\omega t}}{2i} e^{-i2\pi f t} dt \\ &= \frac{1}{2i} \int_{-\infty}^{+\infty} (e^{i\omega t - i2\pi f t} - e^{-i\omega t - i2\pi f t}) dt \\ &= \frac{1}{2i} \int_{-\infty}^{+\infty} (e^{-i2\pi(f - \frac{\omega}{2\pi})t} - e^{-i2\pi(f + \frac{\omega}{2\pi})t}) dt \end{aligned}$$

using the dirac impulse function

$$\delta(f) = \int_{-\infty}^{+\infty} e^{-i2\pi f t} dt$$

we can write $X(f)$ as

$$X(f) = \frac{1}{2i}(\delta(f - \frac{\omega}{2\pi}) - \delta(f + \frac{\omega}{2\pi}))$$

and because $\delta(f)$ has the property of being non-zero only when $f = 0$, we can derive that $X(f)$ is non-zero only when $f = \pm\frac{\omega}{2\pi}$.

Problem 12

The root mean square of a sine wave of amplitude A and frequency f is

$$\begin{aligned} RMS &= \sqrt{f \int_0^{\frac{1}{f}} A^2 \sin^2(2\pi f t) dt} \\ &= A\sqrt{\frac{1}{2} f \int_0^{\frac{1}{f}} (1 - \cos(4\pi f t)) dt} \\ &= A\sqrt{\frac{1}{2} f \Big[t - \frac{1}{4\pi f} \sin(4\pi f t)\Big]_0^{\frac{1}{f}}} \\ &= \frac{A}{\sqrt{2}} \\ &\approx 0.707A \end{aligned}$$