Interdomain Routing as Social Choice

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Abstract

Interdomain routing is essential to both the stability and efficiency of the global Internet. However, most previous studies focus only on stability, and only on a special class of routing protocols, namely BGP-type, path-vector protocols. In this paper, we conduct a systematic analysis of interdomain routing considering optimality and implementation in strategic settings. We adopt the novel perspective that an interdomain routing system is one that defines a social choice rule that aggregates individual preferences of all of the autonomous systems (ASes) in a network to select interdomain routes with a set of desirable properties. An interdomain routing protocol, then, is a mechanism to implement the identified interdomain routing social choice rule, when the ASes can adopt strategic actions. By pointing out the incompatibility among the desirable properties of an interdomain routing system and the requirements for strategic implementation in distributed settings, we reveal fundamental tradeoffs that must be made when extending BGP or designing the next-generation interdomain routing system. We also provide new insights into BGP, by “reverse-engineering” its behaviors from the perspective of social choice and implementation theory.

1. Introduction

Interdomain routing is essential to both the stability and efficiency of the global Internet, which consists of a large number of domains. Since a domain belongs to an independent operator, it has complete freedom in setting its preferences of interdomain routes to reflect its own objectives such as cost optimization, latency reduction, and/or congestion avoidance. To reflect the fact that these domains operate autonomously in the Internet, they are referred to as autonomous systems (ASes). An interdomain routing protocol, together with the preferences of the ASes, produces interdomain routes for the global Internet.

The de facto interdomain routing protocol of the current Internet is BGP, a path-vector protocol. Recent discovery (e.g., [20]) that BGP routing can lead to persistent routing oscillations has led to extensive research on the subject lately. In particular, previous studies (e.g., [5, 10–12, 18]) have investigated general models of BGP-type, path-vector interdomain routing, and identified conditions for stability.

Although the preceding results are surprisingly pleasant and elegant, they consider only a special class of routing protocols, namely BGP-type, path-vector protocols. Even though such studies are clearly important given that BGP is the de facto protocol of the current Internet, a fundamental feasibility study is still missing. In particular, given the increasing concern that BGP is reaching the end of its useful lifetime and that a replacement for BGP is required [2, 4, 19], in order to extend BGP or to design a next-generation interdomain routing protocol, we need to adopt a systematic methodology; that is, we first identify the desirable properties that an interdomain routing system should satisfy, and then investigate the feasibility of designing one satisfying the identified properties, in strategic settings.

A particularly important class of desirable properties of a large-scale system that is important to the society at large are those related with optimality. However, previous studies on interdomain routing focus only on stability. For example, it is unknown whether BGP produces only Pareto optimal routing solutions. Pareto optimality is a central and fundamental requirement in economics, and clearly an interdomain routing protocol which produces only Pareto optimal routing solutions is more preferred to a protocol which does not.

Neither is the implementation of an interdomain routing system in strategic settings thoroughly investigated. Most previous studies on BGP assume that the ASes are obedient; that is, they follow the conventional BGP decision process, where each AS chooses the best available route at any time. In other words, the previous studies mainly follow the traditional view on protocol design which specifies not only the message format, but also the action that each AS should take under each scenario. However, in the context of interdomain routing, since ASes are autonomous and self-optimizing, they may not always follow the specified protocol actions. Instead, based on their preferences, individual
ASes may execute an interdomain routing protocol strategically in order to influence the resulting interdomain routes to their advantage. As a result, when we extend BGP or design a next-generation interdomain routing protocol, it is crucial that we adopt this strategic perspective, so that the interdomain routing protocol actually achieves our goal.

In this paper, we conduct a systematic analysis of interdomain routing considering optimality and strategic implementation properties. Although there are previous studies on designing incentive-compatible mechanisms for interdomain routing (e.g., [6–9, 14, 16, 17]), they all assume inter-AS comparable, quasi-linear utilities. In this paper, we adopt the more general perspective that the objective of interdomain routing is to define a social choice rule which aggregates individual preferences of all ASes in a network to select interdomain routes with a set of desirable properties. An interdomain routing protocol, then, is a mechanism to implement a chosen social choice rule, considering the strategic behaviors of individual ASes.

However, in order to build a social choice model for interdomain routing, we have to overcome a major gap between them. In social choice theory, all players rank the same set of outcomes; while in interdomain routing, the main concern of each AS is the path from itself to a destination. Thus, on the surface, the ASes are ranking different sets of outcomes. Our first major contribution is a new model of interdomain routing that bridges this gap. We make the observation that the set of global routing trees can serve as the unifying common set of outcomes that all ASes rank. Although the notions of routing trees and general preferences on routes have been used to model interdomain routing in previous studies [6, 8], we make the first observation that the preferences of individual ASes on different sets of routes can be converted to preferences on a common set of routing trees, thus enable the application of social choice theory.

This social choice model for interdomain routing leads us immediately to the rich literature of social choice and implementation theory. For excellent surveys on social choice theory, see [1, 13, 15]. These theories provide both methodology for, and insights into the study of interdomain routing. In particular, we show that some requirements to implement an interdomain routing system in strategic settings are incompatible with some highly desirable properties of such a system. These incompatibility results demonstrate the intrinsic challenges in implementing an efficient interdomain routing system in the presence of strategic behaviors, and reveal fundamental tradeoffs that must be made when extending BGP or defining the objective of next-generation interdomain routing.

More importantly, we can now provide new insights into the behaviors of BGP, by “reverse-engineering” its objectives from the social choice perspective. Specifically, we derive the particular social choice rule that is implicitly defined by BGP, when all ASes follow the conventional BGP strategy of picking the best available routes at any time. In particular, We discover that this social choice rule has a surprisingly nice property, naming strong Pareto optimality, under assumptions on the local policies of individual ASes. This result is in sharp contrast with some negative results in general social choice theory. This fact indicates that, due to the special structures of interdomain routing, some classical negative results of social choice theory may not be applicable any more. This fact points us to possible new opportunities to explore when designing the next-generation interdomain routing system.

The rest of the paper is organized as follows. In Section 2, we define our model of interdomain routing from the perspective of social choice theory. In Section 3, we investigate the insights into interdomain routing that can be obtained from this perspective, including a case study of BGP. We summarize our contributions in Section 4.

2. A Social Choice Model for Interdomain Routing

In this section, we shall formulate the objective of interdomain routing as defining a social choice rule (SCR), and an interdomain routing protocol as a mechanism to implement such an SCR. A key observation is that if we consider an interdomain routing system as a black box, then the input to the black box is the local routing policies of all ASes in the network, and the output is various configurations of interdomain routes that may appear under these policies. From this perspective, the objective of interdomain routing is to define the black box, which can be viewed as an SCR that maps the local routing policies of individual ASes to configurations of interdomain routes. This novel perspective enables us to adopt a first-principles approach when studying interdomain routing, in which we first identify a set of desirable properties that the SCR should satisfy, before adding more protocol implementation considerations.

In the sequel, we shall formally define our social choice model for interdomain routing. As preparation, we first introduce some notations.

2.1. Network Topology Definitions and Notations

We represent the network topology by a simple, connected, and undirected graph \( G = (V, E) \), where \( V = \{0, 1, \ldots, N\} \) is the set of ASes, also called players, and \( E \) is the set of interdomain links. Each AS attempts to establish a path to each destination. In this paper, we consider the routing to each destination separately, and let AS 0 be the destination under consideration.
A path in $G$ is either the empty path, denoted by $\epsilon$, or a sequence of ASes $(v_k, v_{k-1}, \ldots, v_1, v_0)$ (for the sake of simplicity, we denote a path by $v_k v_{k-1} \ldots v_0$ in figures), where $k \geq 0$ is the length of the path, such that $(v_i, v_{i-1}) \in E$ for $i = k, k - 1, \ldots, 1$. Note that if $k = 0$, then $(v_0)$ represents the trivial path from $v_0$ to itself. Each nonempty path $P = (v_k, v_{k-1}, \ldots, v_1, v_0)$ has a direction from $v_k$ to $v_0$. If $P$ and $Q$ are two nonempty paths such that the first AS in $Q$ is the same as the last AS in $P$, then $PQ$ denotes the path formed by the concatenation of these two paths. We extend this with the convention that $\epsilon P = P \epsilon = P$ for any path $P$.

### 2.2. Interdomain Routing as Social Choice

Our first step to model interdomain routing as social choice is to identify the set of common outcomes. As we mentioned in the introduction, our key observation here is that routing trees can serve this purpose. Specifically, a routing tree is a directed tree with all edges directed towards the root, AS $0$. Let $T$ be a routing tree, we denote by $T(i)$ the path in it from AS $i$ to AS $0$. For any routing tree $T$, we always have that $T(0) = (0)$. The common set of outcomes, $A$, is a non-empty subset of all possible routing trees that can be formed on the network topology $G$. In general, $A$ can be any non-empty subset of all possible routing trees. This reflects the fact that constraints such as export policies may put restrictions on realizable routing trees. The routing trees in $A$ are also referred to as alternatives.

Our next step in applying social choice is to represent the local routing policies of ASes in terms of preferences over the set $A$. Traditionally, these policies are represented as preferences over sets of routes from each AS to the destination, not the set of routing trees $A$. Nevertheless, a key observation is that preferences over $A$ is a general representation of these policies. Specifically, we map each path $P$ to destination $0$ into a set of routing trees where $P$ is present, and then map for each AS $i$ its local preference (i.e., the ranking for different paths from $i$ to destination $0$) to the ranking of corresponding routing trees in the common set $A$. This way, we essentially convert the preferences of individual ASes on different sets of routes into preferences on the common set of routing trees, namely $A$.

Therefore, in our social choice model of interdomain routing, the local routing policy of AS $i$ is represented as a preference relation, which is a binary relation $R_i$ over $A$ that is complete and transitive. The notion $T_1 R_i T_2$ indicates that $i$ weakly prefers routing tree $T_1$ to $T_2$. The strict preference relation associated with $R_i$ is denoted by $P_i$ (where $T_1 P_i T_2$ if and only if it is not the case that $T_2 R_i T_1$). Also, the indifferent preference relation associated with $R_i$ is denoted by $I_i$ (where $T_1 I_i T_2$ if and only if $T_1 R_i T_2$ and $T_2 R_i T_1$). We denote by $R = (R_1, \ldots, R_N)$ a preference profile, and $(\bar{R}_i, R_{-i})$ the profile where the $i$-th entry of $R$ is replaced with $\bar{R}_i$.

We can now formalize our black-box view of interdomain routing as an SCR as follows.

**Definition 1** A preference domain (or domain for short), $\mathcal{D}$, is a non-empty set of potential preference profiles.

**Definition 2** An interdomain social choice rule (SCR), $F$, is a correspondence mapping a preference profile $R$ in a preference domain $\mathcal{D}$ into a subset of realizable routing trees:

$$F : \mathcal{D} \mapsto 2^A.$$  

When $F$ is single-valued, it is referred to as an interdomain social choice function (SCF).

Note that we require an interdomain SCR to be defined over a whole domain $\mathcal{D}$ of profiles instead of a specific profile. This is because the preferences of individual ASes are unknown, and also likely to change due to changes of factors such as traffic condition. Thus the designer of an interdomain routing system should specify the behavior of the system for not just a single profile of preferences, but all potential preference profiles.

As we will see shortly, the preference domain of an interdomain SCR plays an important role in determining the feasibility of an interdomain routing system. Thus we take this opportunity to introduce the preference domains that are discussed later in this paper.

As in the literature, the most general preference domain is the set of all profiles of complete and transitive preference relations on $A$, and is referred to as the unrestricted domain, $\mathcal{R}_A$. Similarly, the set of all profiles of linear orderings of $A$ is referred to as the unrestricted domain of strict preferences, $\mathcal{P}_A$.

In interdomain routing, there are two preference domains that take into account various constraints in this specific context.

The first domain is the domain of unrestricted route preferences, denoted by $\mathcal{R}_{A}$, which consists of all profiles of unrestricted route preferences on $A$. An unrestricted route preference assumes that an AS cares only about its own path to the destination, and is indifferent between two routing trees in which it has the same path. In other words, a preference relation $R_i$ is an unrestricted route preference (for AS $i$), if for any two routing trees $T_1, T_2 \in A$ such that $T_1(i) = T_2(i)$, we have $T_1 R_i T_2$.

The second domain is the domain of strict route preferences, denoted by $\mathcal{P}_{A}$, which consists of all profiles of strict route preferences on $A$. A strict route preference is a refinement of an unrestricted route preference, and further assumes that an AS always differentiates between two routing trees in which it has different paths. Formally, an unrestricted route preference $R_i$ is a strict route preference
and then computes the outcome(s) using this SCR. Instead, central authority which solicits the preferences of the ASes properties. In the Internet, however, there does not exist a domain routing as defining an SCR with a set of desirable properties are very similar to those in [1], and are omitted here due to space limit.

- **Non-emptiness**: It is clearly desirable that an interdomain SCR selects at least one routing tree for every possible preference profile. Otherwise, instability can occur and some AS may not be able to communicate with the destination, defeating the purpose of routing.

- **Unique outcome**: It is beneficial if an interdomain SCR always chooses a single routing tree, since it removes any potential ambiguity. Generally, due to factors such as symmetry and conflict of interests, an interdomain SCR may choose multiple routing trees. In our framework, we leave choosing any one of them to the interdomain routing protocol. However, if a unique routing tree is achievable, it is clearly more desirable.

- **Unanimity**: It is important that if all ASes agree that one routing tree is the most desirable outcome, then an interdomain SCR should select that tree.

- **Strong Pareto optimality**: Intuitively, if a routing tree \( T_1 \) is less desirable than another routing tree \( T_2 \) by every AS, then \( T_1 \) should not be chosen. Since the Internet is a globally shared resource, it is desirable that interdomain routing only chooses routing trees that are Pareto optimal.

- **Non-dictatorship**: Ideally, one prefers the routing tree chosen by an interdomain SCR to be unanimously acceptable or at least acceptable by a majority of ASes. In real life, however, those ASes closer to the destination may, to a certain extent, have more influence on the final outcome due to their ownership of important topology links near the destination.

2.3. Some Desirable Properties of Interdomain Social Choice

A social choice rule defined by an interdomain routing system should not be arbitrary. We can identify some desirable properties that an interdomain routing system and its SCR should satisfy. The formal definitions of these properties are very similar to those in [1], and are omitted here due to space limit.

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2.4. An Interdomain Routing Protocol as a Mechanism Implementing an SCR

The preceding section formulates the objective of interdomain routing as defining an SCR with a set of desirable properties. In the Internet, however, there does not exist a central authority which solicits the preferences of the ASes and then computes the outcome(s) using this SCR. Instead, the ASes execute an interdomain routing protocol to produce the outcome.

Traditionally, a protocol specifies not only the message format, but also the action that each AS should take under each scenario. However, in the context of interdomain routing, since each AS is autonomous and self-optimizing, it may not always follow the specified protocol action; instead, it may execute the protocol strategically in order to influence the resulting routing tree to their advantage. Therefore, it is best to view an interdomain routing protocol as specifying a set of admissible actions for each AS in every scenario. For instance, BGP specifies only the syntax and semantics of messages exchanged, leaving it open for individual ASes to choose one of the available routes.

In view of this kind of possible strategic behaviors, we take the general perspective that an interdomain routing protocol is actually a game form. For any preference profile, the game form (protocol) induces a game, where individual ASes may choose to execute the protocol strategically to their advantage.

Formally, a game form is a pair \((M, g)\), where \( M = M_1 \times \cdots \times M_N \) is a cross product of message or action spaces and \( g : M \rightarrow A \) is an outcome function. Thus, for each profile of messages \( m = (m_1, \ldots, m_N) \), \( g(m) \in A \) represents the resulting outcome (routing tree). A game form is often also referred to in the literature as a mechanism.

There are configurations of interdomain routes at which we say that the network is at equilibrium. That is, no AS would rationanly change its interdomain route. These equilibria are captured by different solution concepts. Specifically, a solution concept predicts a set of routing tree(s) as outcome(s) resulting from interactions of a certain type of strategic behaviors of ASes.

**Definition 3 (Solution Concept)** A solution concept is a correspondence \( S \) that identifies a subset of \( M \) for any given \((M, g, R)\) specification.

Usually, the specific messages \( S(M, g, R) \) that are predicted by a solution concept are only of intermediate interest as the corresponding set of outcomes is all that we care about. Thus, we pay attention to the outcome correspondence associated with a solution concept \( S \) represented by \( O_S(M, g, R) = \{ a \in A \mid \exists m \in S(M, g, R) \text{ such that } g(m) = a \} \).

**Definition 4 (Implementation)** A social choice rule \( F \) is implemented by the game form \((M, g)\) in the solution concept \( S \) if \( O_S(M, g, R) = F(R) \) for all \( R \in \mathcal{P} \). \( F \) is implementable via the solution concept \( S \) if there exists a game form \((M, g)\) which implements it in \( S \). In particular, implementation via pure Nash equilibrium is referred to as Nash implementation.
3. Implications of the Model

In this section, we explore some of the implications of a social choice model for interdomain routing.

3.1. Some Results from the Literature

The social choice model leads us immediately to the rich literature of social choice and implementation theory. For instance, straightforward applications of some well-known social choice theory results indicate that some requirements to implement an interdomain routing system in strategic settings are incompatible with some highly desirable properties of such a system. Therefore, the model easily reveals fundamental tradeoffs that must be made when extending BGP, or defining the objective of next-generation interdomain routing. The following is a far-from-exhaustive list of such kind of results. The proofs of these results are very similar to those of the corresponding results in [1], and are omitted due to space limit.

Proposition 1 For any network topology with an outcome set $A$, and the unrestricted domain $R_A$, there is no non-empty SCR $F$ over $R_A$ that is non-dictatorial, strategy-proof, and has at least three possible outcomes (routing trees).

Proposition 2 For any given network topology $G$ with corresponding outcome set $A$ and the unrestricted route preference domain $R^*_A$, there is no non-constant SCF that is Nash implementable over $R^*_A$.

Proposition 3 Given a network topology $G$ with the corresponding outcome set $A$ and the unrestricted route preference domain, $R^*_A$, there is no strict Pareto Optimal and non-empty SCR $F$ that is Nash implementable over $R^*_A$.

3.2. A Case Study of BGP

The social choice perspective can also be applied to provide new insights to study a specific protocol, namely BGP. For this purpose, we need to extract the SCR implicitly defined by BGP. Therefore, we abstract away from the protocol-specific details of BGP and model it as a black box that takes routing policies as inputs and outputs a set of interdomain routes. Here, we assume that ASes use the conventional BGP decision process of picking the best route from those available at any time [12]. Since routing policies and interdomain routes can be represented as preference profiles and routing trees respectively, the corresponding SCR, $F_{BGP}$, is just the black-box mapping from preference profiles to sets of routing trees.

We have the following results about $F_{BGP}$. We refer interested readers to [3] for the proofs of these results, which are omitted here due to space limit.

**Proposition 4** Given a network topology $G$ and a corresponding outcome set $A$, $F_{BGP}$ satisfies strong Pareto optimality if the preference domain is the domain of strict route preferences, i.e., $P^*_A$.

Proposition 4 means that whenever BGP converges to a routing tree, it converges to a strongly Pareto optimal one. This is a positive property one would want an interdomain routing protocol to satisfy.

**Proposition 5** Given a network topology $G$ and a corresponding outcome set $A$. Assuming $P$ is the strict route preference domain, $P^*_A$, or the unrestricted route preference domain, $R^*_A$, $F_{BGP}$ is doubly implementable in strong and Nash equilibrium over $P$.

It is worth pointing out that the Nash-implementability of $F_{BGP}$ is not implied by the assumption that each AS always picks the best available route at any time. Actually, Figure 1 shows an example where following this conventional BGP selection process is not a Nash equilibrium.

![Figure 1. An example where following conventional BGP strategy is not a Nash equilibrium.](image)

For the topology given in Figure 1 (a), when each AS always picks the best available route at any time, there are two possible routing trees, $T_1$ and $T_2$ that BGP could converge to. Note that AS 2 prefers $T_2$ over $T_1$. Keeping the strategies of other ASes fixed, AS 2 can adopt the following strategy to always achieve $T_2$:

- if, 30 is available, then select 230;
- if 10 is available, then select 210;
- otherwise select 20.

If AS 2 adopts the above strategy, $T_1$ is no more a stable tree, since AS 2 would pick 210 instead. $T_2$, on the other hand, is still stable. Thus, by deviating from the conventional BGP strategy, AS 2 can achieve a more preferable outcome, which shows that following the conventional BGP strategy is not a Nash equilibrium for this example.

There is an interesting fact implied by the above two results. If we take $P$ to be the sub-domain of $P^*_A$ in which...
interdomain routing system. Opportunities to explore when designing the next-generation results of social choice theory points us to possible new optimality of BGP. The contrast with some classical negative col. A particularly interesting result is the strong Pareto optimality of strategic behaviors. We complemented the above in implementing an efficient interdomain system in the pres-
sibilities demonstrate the intrinsic challenges settings imposes stringent constraints on achievable proper-
ty points us to possible new opportunities to explore when designing the next-generation interdomain routing system.

4 Conclusions

In this paper, we conducted a systematic analysis of interdomain routing considering optimality and strategic implementations. Key observations in the initiation of our study are a black-box view of interdomain routing, and the identification of routing trees as unifying common outcomes. We identified a set of desirable properties that an ideal interdomain routing system should satisfy, and showed that the requirement of implementation in strategic settings imposes stringent constraints on achievable properties. These constraints demonstrate the intrinsic challenges in implementing an efficient interdomain system in the presence of strategic behaviors. We complemented the above general study with an investigation on BGP, a specific protocol. A particularly interesting result is the strong Pareto optimality of BGP. The contrast with some classical negative results of social choice theory points us to possible new opportunities to explore when designing the next-generation interdomain routing system.

References