Abstract

Functional Reactive Programming (FRP) is a paradigm that has simplified the construction of reactive programs. There are many libraries that implement incarnations of FRP, using abstractions such as Applicative, Monads, and Arrows. However, finding a good control flow, that correctly manages state and switches behaviors at the right times, still poses a major challenge to developers.

An attractive alternative is specifying the behavior instead of programming it, as made possible by the recently developed logic: Temporal Stream Logic (TSL). However, it has not been explored so far how Control Flow Models (CFMs), resulting from TSL synthesis, are turned into executable code that is compatible with libraries building on FRP. We bridge this gap, by showing that CFMs are a suitable formalism to be turned into Applicative, Monadic, and Arrowized FRP.

We demonstrate the effectiveness of our translations on a real-world kitchen timer application, which we translate to a desktop application using the Arrowized FRP library Yampa, a web application using the Monadic Threepenny-GUI library, and to hardware using the Applicative hardware description language ClaSH.

CCS Concepts  • Software and its engineering:

Keywords  Reactive Synthesis, TSL, FRP

1 Introduction

Reactive programs implement a broad class of computer systems whose defining element is the continued interaction between the system and its environment. Their importance can be seen through the wide range of applications, such as embedded devices [Helbling and Guyer 2016], games [Perez 2017], robotics [Jing et al. 2016], hardware circuits [Khalimov et al. 2014], GUIs [Czaplicki and Chong 2013], and interactive multimedia [Santolucito et al. 2015].

Functional Reactive Programming (FRP) [Courtney et al. 2003; Elliott and Hudak 1997] is a paradigm for writing programs for reactive systems. The fundamental idea of FRP is to extend the classic building blocks of functional programming with the abstraction of a signal to describe values varying over time. In contrast to sequential programs being executed step by step, FRP programs lead to stream processing networks that manage state and switch between behaviors dependent on the user input. FRP programs can be exceptionally efficient. For example, a network controller recently implemented as an FRP program on a multicore processor outperformed all its contemporary competing implementations [Voellmy et al. 2013].

Building a reactive program is a complex process, of which the most difficult part is finding a good and correct high-level design [Piterman et al. 2006]. Furthermore, even once this design has been fixed, implementing the system still remains a highly error-prone process [Shan et al. 2016]. While FRP helps with the latter problem by embedding the concept of time into the type system, it still leaves the challenge of switching between behaviors and managing state efficiently open. The use of temporal logic has been explored to test properties of FRP programs [Perez and Nilsson 2017], however testing still leaves space for possible errors.

A solution for solving the design challenge has been proposed with Temporal Stream Logic [Finkbeiner et al. 2019], a specification logic to specify the temporal control flow behavior of a program. The logic enforces a clean separation between control and data transformations, which also can be leveraged in FRP [Elliott and Hudak 1997]. Temporal Stream Logic (TSL) is used in combination with a reactive synthesis engine to automatically create an abstract model of temporal control called a Control Flow Model (CFM) satisfying the
yampaButton :: SF (Event MouseClick) Picture
yampaButton = proc click -> do
  rec
    count <- init 0 ->< newCount
    newCount <- arr f1 ->< (click, count)
    pic <- arr f2 ->< count
    returnA ->< pic
f1 :: (Event MouseClick, Int) -> Int
f1 (click, count)
  | isEvent click = count + 1
  | otherwise = count
f2 :: Int -> Picture
f2 = text . show

Figure 1. A button written with the FRP library Yampa.

given specification. TSL combines Boolean and temporal operations with predicates \( p \), evaluated on inputs \( s_1 \), and updates denoted by \( [s_0 \leftarrow f \ s_1] \), which map pure functions \( f \) to inputs \( s_1 \) and pipe the result to an output \( s_0 \). An example for a TSL specification is given by

\[
\square((\text{event click} \rightarrow [\text{count} \leftarrow \text{increment count}]) \land [\text{screen} \leftarrow \text{display count}])
\]

which states that the counter must always be incremented if and only if there is a click event, and that the value of the counter is displayed on a screen.

An implementation that satisfies the specified behavior, built using the FRP library Yampa [Courtney et al. 2003], is shown in Fig. 1. The program implements a button in a GUI which shows a counter value that increments with every click. The Yampa FRP library uses an abstraction called arrows [Hughes 2000], where the arrows define the structure and connection between functions [Liu and Hudak 2007]. As mentioned before, they can be used to cleanly separate data transformations into pure functions, creating a visually clear separation between the control flow and the data level. In the example program of Fig. 1, this separation is clearly visible. The yampaButton is the “arrow” part of the code, which defines a control flow. The functions \( f_1 \) and \( f_2 \) are the “functional” part, describing pure data transformations.

In the TSL specification, function applications, like click, increment or display, are not tied to a particular implementation. Instead, the synthesis engine ensures that the specification is satisfied for all possible implementations that may be bound to these placeholders, similar to an unknown polymorphic function that is used as an argument in a functional program. Thus, the implementation of Fig. 1 indeed satisfies the given specification by implementing event with isEvent of the yampa library, increment with \((+1)\), and display with text of the glossary library and show.

The immediate advantage of synthesis over manual programming is that, if the synthesis succeeds, then there is a guarantee that the constructed program satisfies the specification. Sometimes, the synthesis does not succeed, and this also leads to interesting results. An example is given by the FRPZoo [Gélineau 2016] study, which consists of implementations for the same program for 16 different FRP libraries. The program consists of two buttons that can be clicked on by the user: a counter button, which keeps track of the number of clicks, and a toggle button, which turns the counter button on and off. To our surprise, after translating the written-English specification from the FRPZoo website into a formal TSL specification, the synthesis procedure was not able to synthesize a satisfying program. By inspecting the output of the synthesis tool, we noticed that the specification is actually unrealizable. The problem is that the specification requires the counter to be incremented whenever the counter button is clicked. At the same time, the counter must be reset to zero whenever the toggle button is clicked. This creates a conflict when both buttons are clicked together. To obtain a solution, we had to add the assumption that both buttons are never pressed simultaneously.

While the work of Finkbeiner et al. 2019 discusses the synthesis process for creating the CFM in detail, it does not elaborate on how a CFM is actually turned into FRP code, which is necessary to finally generate an executable. In this work we explore this process, and show how Causal Commutative Arrows (CCA) form a foundational abstraction for FRP in the context of program synthesis. From this connection between CCA and a CFM, we build a system to generate library-independent FRP across a range of FRP abstractions.

There is no single style of FRP which is generally accepted as canonical. Instead, FRP is realized through a number of libraries, which are based on fundamentally different abstractions, such as Monadic FRP [Apfelmus 2013; Ploeg and Claessen 2015; Trinkle 2017], Arrowized FRP [Courtney et al. 2003; Winograd-Cort 2015], and Applicative FRP [Apfelmus 2012; Baaij 2015].

We show that our system is flexible enough to handle all of these abstractions, by demonstrating translations from a CFM to Threepenny-GUI [Apfelmus 2013], Yampa [Courtney et al. 2003], and ClaSH [Baaij 2015]. The CFM used to generate the code is synthesized from a TSL specification that describes the behavior of a kitchen timer application. It was obtained from a real-world kitchen timer, as depicted in Fig. 2, that is retailed by the German company Dirk Rossmann GmbH.

We do not envision FRP synthesis as a replacement for FRP libraries, but rather as a complement. Through synthesis and code generation, users automatically construct FRP programs in these libraries that provide critical interfaces to input/output domains. Furthermore, we show that TSL synthesis generates code as expressive as CCA. While this power is sufficient for many applications, the FRP libraries still provide an interface to more powerful language abstraction features, in case the user chooses to use them.
In summary, the paper makes the following contributions:

1. We describe the process of automatically generating library-independent FRP control code from TSL specifications.
2. We examine the relation between CFMs and CCA, and compare the differences between various FRP abstractions during the translation process.
3. We demonstrate our translations on a kitchen timer application, built as a desktop application using the Arrowized FRP library Yampa, as a web application using the Monadic library Threepenny-GUI, and to hardware using the Applicative hardware description language ClaSH.
4. We provide an open-source tool for the synthesis of FRP programs from TSL specifications.

2 Preliminaries

We assume time to be discrete and denote it by the set $\text{Time}$ of positive integers. A value is an arbitrary object of arbitrary type, where we use $\mathcal{V}$ to denote the set of all values. We consider the Boolean values $\mathcal{B} \subseteq \mathcal{V}$ as a special subset, which are either $\text{true} \in \mathcal{B}$ or $\text{false} \in \mathcal{B}$.

A signal $s : \text{Time} \rightarrow \mathcal{V}$ is a function fixing a value at each point in time. The set of all signals is denoted by $\mathcal{S}$, usually partitioned into input signals $I$ and output signals $O$.

An $n$-ary function $f : \mathcal{V}^n \rightarrow \mathcal{V}$ determines a new value from $n$ given values. We denote the set of all functions (of arbitrary arity) by $\mathcal{F}$. Constants are functions of arity zero, while every constant is also a value, i.e., an element of $\mathcal{F} \cap \mathcal{V}$.

An $n$-ary predicate $p : \mathcal{V}^n \rightarrow \mathcal{B}$ checks a truth statement on $n$ given values. The set of all predicates (of arbitrary arity) is denoted by $\mathcal{P}$.

2.1 Functional Reactive Programming

FRP is a programming paradigm that uses the abstractions available in functional programming to create an abstraction of time. The core abstraction in FRP is that of a signal

$$\text{Signal } a = \text{Time} \rightarrow a$$

which produces values of some arbitrary type $a$ over time. The type $a$ can be an input from the world, such as the current position of the mouse, or an output type, such as some text that should be rendered to the screen. Signals are also used internally to manipulate values over time, for example if the position of the mouse should be rendered to the screen.

Arrows There are many incarnations of FRP, which use various abstraction to manipulate signals over time. One popular abstraction for FRP is a Monad, but a weaker abstraction, called Arrows, is also used in many modern libraries [Murphy 2016; Perez et al. 2016]. The Arrow abstraction describes a computation connecting inputs and outputs in a single type [Hughes 2000]. Hence, an Arrow type also allows us to describe reactive programs that process inputs and produce outputs over time.

Arrowized FRP was introduced to plug a space leak in the original FRP work [Elliott and Hudak 1997; Liu and Hudak 2007]. By using the Arrow abstraction introduced in [Hughes 2000], which describes in a single type inputs and outputs, we can also describe reactive programs that process inputs and produce outputs over time. At the top level, an Arrowized FRP program will have the form

$$SF \text{ Input Output } = \text{Signal Input } \rightarrow \text{Signal Output}$$

which is a signal function type, parametrized by the type of input from the world and the type of output to the world. The core Arrow operators, shown in Fig. 3, are used to compose multiple arrows into larger programs.
### 2.3 Connections between FRP and Reactive Systems

A first inspection reveals that FRP fits into the definition of a reactive system, as given in Section 2.2: an FRP program reads an infinite stream of input signals and finally produces a corresponding infinite output stream. Nevertheless, FRP does not fit into the classical setting used for reactive systems, as the input and output streams in FRP are allowed to have arbitrary types.

To solve this problem, one could restrict FRP to just streams of enumerative types, which then are reduced to a Boolean representation. However, this would drop the necessity of almost all interesting features of FRP and it is questionable whether this restricted notion of FRP would give any benefits against Mealy/Moore automata or circuits, which are already used for reactive systems. Additionally, it just creates an exponential blowup and does not provide any insights into the core problem.

Hence, it is more reasonable and interesting to ask whether it is possible to natively handle streams of arbitrary type within reactive systems. Recall that FRP includes functional behavior, defined using standard functional paradigms, but also a control structure, defined via arrows and loops. We will target synthesis of the control structure, leaving the functional level synthesis to tools such as Myth [Kuncak et al. 2010; Osera and Zdancewic 2015] or Synquid [Polikarpova et al. 2016].

### 2.4 Temporal Stream Logic

We use Temporal Stream Logic (TSL) for specifying the control flow behavior of functional reactive programs [Finkbeiner et al. 2019]. TSL has been especially designed for synthesis and describes control flow properties with respect to their temporal behavior. If a TSL specification is realizable it can be turned into a Control Flow Model (CFM), an abstract representation of the FRP network that covers all possible behavior switches.

Temporal Stream Logic builds on the notion of updates, such as \([y \leftarrow f x]\), which expresses that the result of mapping the pure function \(f\) to some input stream \(x\) is piped to the output stream \(y\). The execution of an update is coupled with predicate evaluations guiding the control flow decisions of the network. In combination with Boolean and temporal operators, the logic allows for expressing even complex temporally evolving FRP networks using only a short, but precise description of the temporal behavior.

TSL specifications implicitly induce an architecture as shown in Fig. 5. As a basis, the syntax of TSL uses a term based notion, built from input streams \(i \in I\), output streams \(o \in O\), memory cells \(c \in \mathbb{C}\), and function and predicate literals \(f \in F\) and \(p \in P\) with \(P \subseteq F\), respectively. The purpose of cells is to memorize data values, output to a cell at time \(t \in \text{Time}\), to provide them again as inputs at time \(t + 1\).
We differentiate between function terms $\tau_F \in \mathcal{T}_F$ and predicate terms $\tau_P \in \mathcal{T}_P$, built according to the following grammar:

$$
\begin{align*}
\tau_F & := s_1 | f \tau_F^0 \tau_F^1 \cdots \tau_F^{n-1} \\
\tau_P & := p \tau_P^0 \tau_P^1 \cdots \tau_P^{n-1}
\end{align*}
$$

where $s_1 \in \mathcal{I} \cup \mathcal{C}$ is either an input stream or a cell. In a TSL formula $\phi$, predicate and function terms are then combined to updates with Boolean connectives and temporal operators (as well as the standard derived Boolean and temporal operators).

$$
\phi := \tau_P \mid \big[ s_o \leftarrow s \big] \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \mathcal{U} \phi
$$

where $s_o \in \mathcal{O} \cup \mathcal{C}$ is either an output signal or a cell.

For giving semantics to TSL formulas $\phi$, a universally quantified assignment function $\langle \cdot \rangle : \mathcal{R} \rightarrow \mathcal{F}$ is used. The assignment function fixes an implementation for each predicate and function literal, as well as input streams $i : I \rightarrow \mathcal{S}$.

We will only give an intuitive description of the semantics here. For a fully formal definition, the interested reader is referred to [Finkbeiner et al. 2019]. Intuitively, the semantics of TSL can be summarized as follows:

**Predicate terms** are evaluated to either true or false, by first selecting implementations for all function and predicate literals according to $\langle \cdot \rangle$, and then applying them to the inputs, as given by $i$, and cells, using the stored value at the current time $t$. The content of a cell thereby is fixed iteratively by selecting the past values piped into the cell over time. Cells are initialized using a special constant provided as part of $\langle \cdot \rangle$.

**Function terms** are evaluated similar to predicate terms, except that they can evaluate to any value of arbitrary type.

**Updates** are used to pipe the result of function term evaluations to output streams or cells. Updates, as they appear in a TSL formula, are typed as Boolean expressions. In that sense, update expressions are used in TSL formulas to state that a specific flow is executed at a specific point in time, where the expression evaluates to true, if it is executed and to false, otherwise. The semantics of TSL ensure that different updates to the same output or cell are always mutually exclusive, e.g., the expression $\big[ o \leftarrow f \ x \big] \land \big[ o \leftarrow g \ x \big]$ is never satisfied.
We demonstrate the advantages of using TSL as a specification language for the development of FRP applications using the example application of a kitchen timer, as presented in Section 3 System Design with TSL.

### 3 System Design with TSL

We demonstrate the advantages of using TSL as a specification language for the development of FRP applications using the example application of a kitchen timer, as presented in Section 3 System Design with TSL.

![Diagram of a kitchen timer](image)

**Figure 7.** The kitchen timer concept.

1. Whenever the MIN and SEC buttons are pressed simultaneously, the timer is reset, meaning the time is set to zero and the system stays idle until the next button gets pressed.
2. If only the MIN button is pressed and the timer is not currently counting up or down, then the currently set time is increased by one minute.
3. If only the SEC button is pressed and the timer is not currently counting up or down, then the currently set time is increased by one second.
4. As long as no time greater than zero has been set and the system is idle: if the START/STOP button is pressed and the timer is not already counting up or down, then the timer starts counting up until it is stopped by any button pressed.
5. If a time has been set and the START/STOP button is pressed while the timer is not currently counting up or down, then the timer starts counting down until it is stopped by any button pressed.
6. The timer can only be started by pressing start.
7. The timer can always be stopped by pressing any button while counting up or down.
8. It is possible to start the timer and to set some time simultaneously.
9. The buzzer beeps on any button press and after the counter reaches zero while counting down.
10. The display always shows the time currently set.

While it requires a certain amount of engineering for finding the right control behavior, especially for fixing the additional state to manage the different modes, when directly implementing the application on top of an FRP library, it is an easy task to specify the control behavior with TSL. We first fix possible operations on time, used as a cell for holding the currently set time.

\[
\text{COUNTUP} := [\text{time} \leftarrow \text{countup time} \ dt]
\]

\[
\text{COUNTERDOWN} := [\text{time} \leftarrow \text{countdown time} \ dt]
\]

\[
\text{INCMIN} := [\text{time} \leftarrow \text{incMinutes time}]
\]

\[
\text{INCSEC} := [\text{time} \leftarrow \text{incSeconds time}]
\]

\[
\text{IDLE} := [\text{time} \leftarrow \text{time}]
\]

The used literals countup, countdown, incMinutes, and incSeconds represent pure functions that update the value of time accordingly, while the input signal dt delivers the time difference since the last execution of the network. By the semantics of TSL it is already ensured that assignments to the same cell are mutually exclusive, i.e., it can never be the case that time is counting up and the minutes are increased at the same time.

Next, we fix the control flow behavior of time. In our case, we need a predicate to check whether the time currently set is zero or not

\[
\text{ZERO} := \text{eq time zero}
\]

where zero is a constant function of the same type as time. We also fix some sub-properties, that are useful to express conditions regularly appearing in the main specification later. In our case these are

\[
\text{RESET} := \text{btnMin} \land \text{btnSec}
\]

\[
\text{COUNTING} := \text{COUNTUP} \lor \text{COUNTERDOWN}
\]

\[
\text{ANYKEY} := \text{press btnMin} \lor \text{press btnSec} \lor \text{press btnStartStop}
\]

\[
\text{START} := \text{press btnStartStop} \land \lnot \text{press btnMin} \land \lnot \text{press btnSec}
\]

\[
\text{START} \& \text{MIN} := \text{press btnStartStop} \land \text{press btnMin} \land \bigcirc (\lnot \text{btnSec} \land \lnot \text{btnMin})
\]

\[
\text{START} \& \text{SEC} := \text{press btnStartStop} \land \text{press btnSec} \land \bigcirc (\lnot \text{btnMin} \land \lnot \text{btnSec})
\]

The literals btnMin, btnSec, and btnStartStop represent the input signals for the three buttons, respectively. The function press is used as a helper function for improved readability and is defined as

\[
\text{press x} := \lnot x \land \bigcirc x
\]

The additional conditions \text{START} \& \text{MIN} and \text{START} \& \text{SEC} are used for realizing requirement 8. This is all we need for implementing the invariants of the aforementioned requirements:

\[
\psi_1 := \text{RESET} \leftrightarrow [\text{time} \leftarrow \text{zero}]
\]

\[
\psi_2 := \lnot \text{COUNTING} \land \text{press btnMin} \land \bigcirc \lnot \text{btnSec} \leftrightarrow \bigcirc \text{INCMIN}
\]

\[
\psi_3 := \lnot \text{COUNTING} \land \text{press btnSec} \land \bigcirc \lnot \text{btnMin} \leftrightarrow \bigcirc \text{INCSEC}
\]

\[
\psi_4 := \text{ZERO} \rightarrow (((\text{IDLE} \land \text{START}) \rightarrow \bigcirc \text{tillAnyInput COUNTUP}) \quad \bigcirc (\text{INCMIN} \lor \text{INCSEC}))
\]

\[
\psi_5 := \text{INCMIN} \lor \text{INCSEC} \rightarrow (\lnot \text{COUNTING} \land \text{START} \rightarrow \bigcirc \text{tillAnyInput COUNTDOWN}) \quad \bigcirc (\text{INCMIN} \land \text{INCSEC})
\]

\[
\psi_6 := \lnot \text{COUNTING} \land \text{O} \land \text{COUNTING} \rightarrow \bigcirc \text{START} \lor \text{START} \& \text{MIN} \lor \text{START} \& \text{SEC}
\]

\[
\psi_7 := \text{COUNTING} \land \text{ANYKEY} \land \lnot \text{RESET} \rightarrow \bigcirc \text{tillAnyInput IDLE}
\]

\[
\psi_8 := \lnot \text{COUNTING} \land (\text{START} \& \text{MIN} \lor \text{START} \& \text{SEC}) \rightarrow \bigcirc \bigcirc \text{tillAnyInput COUNTDOWN}
\]

\[
\psi_9 := (((\text{beep} \leftrightarrow \text{true}) \oplus ((\text{beep} \leftrightarrow \text{false})) \land (\bigcirc (\text{COUNTDOWN} \land \text{ZERO}) \land \text{ANYKEY} \leftrightarrow \bigcirc (\text{beep} \leftrightarrow \text{true}))))
\]

\[
\psi_{10} := [\text{screen} \leftarrow \text{display time}]
\]

The function tillAnyInput is a helper function to denote that a condition must be satisfied until either the system is reset or any button gets pressed. It is defined as:

\[
\text{tillAnyInput } x := (x \land \lnot \text{ANYKEY}) \quad \bigcirc (\text{RESET} \lor x \land \text{ANYKEY})
\]

The final specification is given by \( \phi = \psi_{\text{init}} \land \bigcirc \bigwedge_{i=1}^{10} \psi_i \), where \( \psi_{\text{init}} \) adds some remaining initial conditions. The full specification, using our plain text specification format, is also given in Fig. 8. Note that the various \text{O} operations in the formulas \( \psi_i \) are always necessary, since the "being pressed" condition requires a change of the input, which is only observable by comparing the currently provided value with the previous one.

For the development of such specifications, the designer also gets feedback from the synthesis engine. For example, the condition of \( \psi_2 \) requires btnSec to not be pressed in order to increase the minutes of the counter. Without this condition, the synthesis engine would return an unrealizability result, since increasing minutes would conflict with setting the time to zero on a potential reset, which also requires btnMin to be pressed.
COUNTUP = [ time <- countup time dt ];
COUNTDOWN = [ time <- countdown time dt ];
INCMIN = [ time <- incMinutes time ];
INCSEC = [ time <- incSeconds time ];
IDLE = [ time <- time ];
ZERO = eq time zero();
RESET = btnMin && btnSec;
COUNTING = COUNTUP || COUNTDOWN;
ANYKEY = press btnMin ||
   press btnSec || press btnStartStop;
START = press btnStartStop &&
   !press btnSec && !press btnMin &&
   press btnMin && X ! btnSec && X X ! btnSec;
STARTANDMIN = press btnStartStop &&
   press btnMin && X ! btnSec && X X ! btnSec;
STARTANDSEC = press btnStartStop &&
   press btnSec && X ! btnMin && X X ! btnMin;
xor x y = !(x <-> y);
ZERO = eq time zero();
RESET = btnMin && btnSec;
COUNTING = COUNTUP || COUNTDOWN;
ANYKEY = press btnMin ||
   press btnSec || press btnStartStop;
START = press btnStartStop &&
   !press btnSec && !press btnMin &&
   press btnMin && X ! btnSec && X X ! btnSec;
STARTANDMIN = press btnStartStop &&
   press btnMin && X ! btnSec && X X ! btnSec;
STARTANDSEC = press btnStartStop &&
   press btnSec && X ! btnMin && X X ! btnMin;
xor x y = !(x <-> y);
press x = !x && X x;
tillAnyInput x =
   (x && !ANYKEY) W (RESET || x && ANYKEY);
 initially guarantee {
   [ COUNTING && (X COUNTING -> START);
   INCSEC && ! INCMIN;
   [ beep <- false ];
   }
always guarantee {
  RESET
  <-> [ time <- zero() ];
  [ COUNTING && press btnMin && X ! btnSec
  <-> X INCMIN;
  [ COUNTING && press btnSec && X ! btnMin
  <-> X INCSEC;
  ZERO
 etSocketAddress [ time <- COUNTUP ],
  W (INCMIN || INCSEC);
  INCMIN || INCSEC -> ([COUNTING && START
  -> X tillAnyInput COUNTDOWN) W X ZERO];
  COUNTING && ANYKEY && X ! RESET
  -> X tillAnyInput IDLE;
  [ COUNTING && (STARTANDMIN || STARTANDSEC
  -> X X tillAnyInput COUNTDOWN;
  X ! COUNTING && X COUNTING
  -> X START || STARTANDMIN || STARTANDSEC;
  X (COUNTDOWN && ZERO) || ANYKEY
  <-> X [ beep <- true ];
  xor [ beep <- true ] [ beep <- false ];
  [ dsp <- display time ];
}
control
:: Applicative signal
⇒ _ –– FRP abstraction
⇒ _ –– cell implementation
→ (_ → _) –– functions and predicates
→ _ –– initial values
→ signal _ –– input signals
→ signal _ –– output signal

(a) The general template of the type signature for the control block of the synthesized FRP program.

control
:: Applicative signal
⇒ (V p. p → signal p → signal p)
⇒ (a → Bool) –– press
⇒ (b → c) –– display
⇒ (b → b) –– increment
⇒ b –– initial value: count
⇒ c –– initial value: screen
→ signal a –– button (input)
→ ( signal b –– count (output)
, signal c –– screen (output)
)

(b) An example instantiation of the type signature for the control block of the button (as described in the introduction) as it has been specialized for Applicative FRP.

Figure 10. The control block follows a general type signature template across FRP abstractions.

Given a CFM that satisfies the TSL specification, we convert it into a template for our FRP program. The code implementing the CFM is given in Fig. 9. The CFM is transformed via a syntactic transformation into an FRP program in the abstraction of the user’s choice, as a function that is parameterized over the named function and predicate terms, as shown in Fig. 10. The user then provides implementations of the function and predicate terms that complete the construction of the FRP program based on the generated template.

The CFM transformation is modularized to fit any FRP library that is at least as powerful as CCA [Gélineau 2016; Murphy 2016; Patai 2010; Perez et al. 2016; Floeg and Claessen 2015]. The key insight is that first-order control along with a loop describes the expressive power of both CCA and the CFM model generated from the techniques of [Finkbeiner et al. 2019]. Although many FRP libraries support more powerful operations than CCA, e.g., switch in Yampa, we do not need to utilize these in the synthesis procedure, and thus can generalize synthesis to target any FRP library that is at least as expressive as CCA.

Recall that in TSL, output signals can be written at the current time \( t \) and be read from at time \( t + 1 \). To implement this in the FRP program, we use the concept of a cell in the CFM. In the translation, we allow a space for the user to provide an implementation of the cell that is specialized to their FRP library of choice (as shown in Fig. 10). In the case of CCA, this is the \( \text{loop}\Phi \) combinator. The \( \text{loop}\Phi \) combinator pipes the output values back to the input to allow them to be read at time \( t + 1 \). Since a system may require output values at \( t = 0 \), the user must also provide initial values to \( \bigcirc \).

4.1 Properties
In the translation of the CFM, we use Casual Commutative Arrows (CCA) as the target conceptual model. Understanding the implications of using CCA as an underlying model to connect TSL to FRP allows us to gain insight into the expressive power and limitations of using TSL synthesis to construct FRP programs. One interesting note about this is that CCA does not allow the \text{arrowApply} function, enforcing a static structure on the generated program. The \text{arrowApply} (also called \text{switch}) function is a higher-order arrow that allows for dynamically replacing an arrow with a new one that arrives on an input wire. While \text{switch} is a very expressive operation, it also comes with drawbacks. First, dynamically evolving networks cannot provide runtime guarantees for memory requirements in general, while static networks do. Second, the behavior of a dynamically evolving network is hard to grasp in general, which especially makes them unamenable for verification. Third, the use of dynamic networks is largely impractical for FRP applications with restricted resources, as for example applications that are executed on embedded devices [Sawada and Watanabe 2016] or are implemented directly in hardware [Baaij 2015]. An insight provided by prior work on CCA [Liu et al. 2011] was that, in general, the expressive power of higher-order arrows makes automatic optimization more difficult. Furthermore, for most FRP programs, first-order switch is more than enough [Winograd-Cort and Hudak 2014].

For a full description of the formal properties of TSL synthesis, see the work of [Finkbeiner et al. 2019]. In summary the synthesis procedure is sound, but not complete. From a programming languages design perspective, this means that “compilation” (synthesis) of a specification may not terminate, but when it does terminate, it will generate code that satisfies the specification.

With respect to the synthesis procedure, this is a fundamental restriction related to TSL. With TSL, every update term \( \boxed{x \leftarrow y} \) is lifted to an arrow that updates \( x \) with \( y \) over time. Since in TSL updates are fixed by the specification, so too must the arrow structure be fixed in synthesis. Note that having a fixed arrow structure disallows higher-order arrows, but higher-order functions can still be passed along wires. As an example, we may have a function term \( \text{app} : (a \rightarrow b) \rightarrow a \rightarrow b \) and signals \( f : a \rightarrow b \) and \( x : a \). A simple specification making use of higher order functions then could state that the system should always apply the incoming higher-order function to the incoming value: \( \boxed{[x \leftarrow \text{app}(f, x)]} \).
control :: (MonadFix monad, Applicative signal) ⇒ (∀ p. p → signal p → monad (signal p))
      ...
      → signal a → monad (signal b, signal c)

(a) The Monadic control for the button from the introduction.

control :: (Arrow signal, ArrowLoop signalfunction) ⇒ (∀ p. p → signalfunction p p))
      ...
      → signalfunction a (b, c)

(b) The Arrowized control for the button from the introduction.

control :: HiddenClockReset domain gated synchronous
      ...
      → Signal dom a → (Signal dom b, Signal dom c)

(c) The Applicative (specialized to ClasSH) control for the button from the introduction.

Figure 11. The abbreviated type signatures of the synthesized control blocks for each FRP framework abstraction.

Additionally, a key difference between arrows and circuits is that arrows are able to carry state that tracks the application of each arrow block. By using CCA, a user may write TSL specifications about stateful arrows that are still handled correctly by the synthesis procedure. To this end, we only synthesize programs that obey the commutativity law [Liu et al. 2011; Yallop and Liu 2016] restated below that ensures that arrows cannot carry state influencing the result of composed computations.

\[ \text{first } f \gg \text{second } g = \text{second } g \gg \text{first } f \]

Imagine an arrow with a global counter to track data of a buffer. Since addition is commutative, this arrow respects the commutativity law. However, non-commutative state is possible as well. For example, when building GUIs with Arrowized FRP [Winograd-Cort 2015], the position of each new UI element depends on the order of the previously laid out elements. Due to the commutativity of the Boolean operators, the commutativity of CCA is a necessary precondition for synthesis of a TSL specification. Specifically, the commutativity of logical conjunction allows the solution to update signals in any arbitrary order. Thus, the correctness of the TSL synthesis relies on commutativity of composition, which is naturally modeled with CCA’s commutativity law.

4.2 Example: Kitchen Timer

We revisit the Kitchen Timer application introduced in Section 3 to show the concrete code that is generated. From the TSL specification, we first generate a CFM using our TSL synthesis toolchain together with the LTL synthesizer

\text{strixx} [Meyer et al. 2018] [version 18.04]. The resulting CFM utilizes six additionally synthesized cells and consists of 1188 vertices. This CFM is then transformed into a control structure for each of the different application domains. In Fig. 11, we show how the template described in Section 4 is specialized to each of the three application domains: the desktop program is built with Yampa [version 0.13] and the web app with Threepenny-GUI [version 0.8.3.0]. Both have been built using stack\(^2\) on \texttt{lts-13.17 [ghc-8.6.4]. For building the hardware implementation, we first use the functional hardware description language ClasSH\(^3\) to generate verilog-code, which then is turned into the \texttt{blif} format using the open synthesis suite yosys\(^4\). Afterwards, the generated \texttt{blif}-file is placed using the place-and-route tool \texttt{nextpnrr}\(^5\). The packaged result is then uploaded to an \texttt{iCEblink40HX1K} Evaluation Kit Board from Lattice Semiconductor, featuring an \texttt{ICE40HX1K} FPGA with 100 IO-pins and 1280 logic cells, additionally equipped with the required hardware components. The interfaces to the corresponding timer applications are depicted in Fig. 12.

Note that synthesis only needs to be executed once. Afterwards, code is generated from the resulting CFM and combined with function implementations and initialization

\(^2\)\url{https://www.haskellstack.org} 
\(^3\)\url{https://github.com/clash-lang/clash-compiler [commit: fff4606]} 
\(^4\)\url{https://github.com/YosysHQ/yosys [commit: 70df18]} 
\(^5\)\url{https://github.com/YosysHQ/nextpnr [commit: 5344bc3]}
Table 1. Synthesis and compilation times for creating the different timer applications from the TSL specification.

<table>
<thead>
<tr>
<th>Executed Tool</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synthesis → strix</td>
<td>4.965</td>
</tr>
<tr>
<td>Compilation</td>
<td></td>
</tr>
<tr>
<td>Desktop → Yampa</td>
<td>19.403</td>
</tr>
<tr>
<td>Web → Threepenny-GUI</td>
<td>18.344</td>
</tr>
<tr>
<td>Hardware</td>
<td></td>
</tr>
<tr>
<td>→ ClaSH</td>
<td>11.218</td>
</tr>
<tr>
<td>→ yosys</td>
<td>6.405</td>
</tr>
<tr>
<td>→ nextpnr</td>
<td>7.276</td>
</tr>
</tbody>
</table>

procedures for each of the frameworks. Both, the desktop and the web application only require GHC for compilation, while for hardware, we need multiple translation steps. The respective synthesis and compilation times of the different tools are depicted in Table 1. The full code of the Arrow module, generated from the CFM realizing the specification of the introduction, is given as an example in Fig. 13. The specification, the building framework, and a list of the required hardware components can be found at:

https://github.com/reactive-systems/KitchenTimer

Yampa uses Arrowized FRP which easily fits into the general interface for Arrows that we provide (Fig. 11b). Likewise, Threepenny-GUI uses a Monadic FRP (where the signals themselves are Applicative) which also easily fits into our general interface for Monadic FRP (Fig. 11a). Finally, for ClaSH, we use a mostly Applicative interface, that is specialized to handle the peculiarities of hardware (which needs explicit clocks, as opposed to more traditional FRP frameworks). If we wanted to support other libraries with explicit clocks, for example, as presented in [Bärenz and Perez 2018], we would need a specialized module - although the customization is limited mostly to the type signature generation as shown in Fig. 11c.

Each control block requires the user to provide a cell implementation. Both Yampa and ClaSH provide native implementations of the concepts, as shown in Fig. 14. Although Threepenny-GUI does not provide the exact implementation of a cell, as we require in our synthesized control block, it can be easily implemented using the available primitives of the library.

5 Related Work

There are various lines of work that are related to our approach. While we draw inspiration from these research directions, each one, on its own, addresses a different type of problem.

Figure 13. Generated Arrow code for the intro example.
In contrast, we use the logic TSL, for a fine-grained description of function application behavior which cannot be expressed within pure LTL. The synthesis procedure of TSL determines a temporal control flow of functions, where the TSL specifications determines the transformations to be applied at each point in time. In addition to the logical specifications, the synthesis is also constrained by the types of functions appearing in the specification. Since the types of all functions are fixed at all times, the type system can be lifted to the specification. If the specification is well typed, synthesis is guaranteed to yield a well typed program.

One connection to our work however is the implications of the fact that the Curry-Howard correspondence extends to FRP and LTL. In the aforementioned work, LTL propositions are types for FRP programs. If a proof of a TSL proposition can be interpreted as a program, one might expect that there is some corresponding type system to TSL. We leave such explorations to future work.

5.2 Synthesis of Reactive Programs

A distinguishing feature of our approach is the connection to an actual programming paradigm, namely FRP. Most reactive synthesis methods instead target transition systems or related formalisms such as finite state machines. The idea to synthesize programs rather than transition systems was introduced in [Madhusudan 2011]. In his work, an automaton is constructed that works on the syntax tree of the program, which makes it possible to obtain concise representations of the implementations, and to determine how many program variables are needed to realize a particular specification. Unlike our FRP programs, Madhusudan’s programs only support variables on a finite range of instances.

Another related approach is the synthesis of synchronization primitives introduced in [Bloem et al. 2012] for the purpose of allowing sequential programs to be executed in parallel. Similar to our synthesis approach, uninterpreted functions are used to abstract from implementation details. However, both the specification mechanism (the existing program itself is the specification) and the type of programs considered are completely different from TSL and FRP.

5.3 Logics for Reactive Programs

Many logics have been proposed to specify properties of reactive programs. Synthesis from Signal Temporal Logic [Raman et al. 2015] focuses on modeling physical phenomena on the value level, introducing continuous time and resolving to a system of equations. The approach allows for different notion of data embedded into the equations. While more focused on the data level, the handling of continuous time might provide inspiration for future extensions to explicitly handle continuous time.

Another logic that has been proposed, Ground Temporal Logic [Cyrluk and Narendran 1994], is a fragment of First Order Logic equipped with temporal operators, where it is

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Figure 14. The implementations for a cell in the CFM is commonly found across FRP libraries, or easily re-implemented.
not allowed to use quantification. Satisfiability and validity problems are studied, with the result that only a fragment is decidable. However, specifications expressed in Ground Temporal Logic, as well as their motivations, are completely different from our goals.

5.4 Reasoning-based Program Synthesis

Reasoning-based synthesis [Kuncak et al. 2010; Osera and Zdancewic 2015; Solar-Lezama 2013; Vechev et al. 2013] is a major line of work that has been mostly, but not entirely, orthogonal to reactive synthesis. While reactive synthesis has focused on the complex control aspects of reactive systems, deductive and inductive synthesis has been concerned with the data transformation aspects in non-reactive, sequential programs. Our work is most related to Sketching [Solar-Lezama 2015]. In Sketching the user provides the control structure and synthesizes the transformations while, in TSL we synthesize the control and leave the transformations to the user.

The advantage of deductive synthesis is that it can handle systems with complex data. Its limitation is that it cannot handle the continuous interaction between the system and its environment, which is typical for many applications, such as for cyber-physical systems. This type of interaction can be handled by reactive synthesis, which is, however, typically limited to finite states and can therefore not be used in applications with complex data types. Abstraction-based approaches can be seen as a link between deductive and reactive synthesis [Beyene et al. 2014; Dimitrova and Finkbeiner 2012].

Along the lines of standard reactive synthesis, our work is focused on synthesizing control structures. We extend the classic approach by also allowing the user to separately provide implementations of data transformations. This is useful in the case where the value manipulations are unknown or beyond the capability of the synthesis tool. For example, a user may want to synthesize an FRP program that uses closed source libraries, which may not be amenable to deductive synthesis. In this case, the user can only specify that certain functions from that API should be called under certain conditions, but cannot and may not want to reason about their output.

6 Conclusions

In this work we have presented a detailed account of how to transform Control Flow Models into framework-independent FRP code. With this transformation, we utilize TSL synthesis as presented in [Finkbeiner et al. 2019] to build a complete toolchain for synthesizing Functional Reactive Programs. Using TSL specifications prior to manually programming improves designing the underlying control flow. The developer is immediately notified about conflicts in the current design and supported by the feedback returned from synthesis for resolving them.

So far, we have used a discrete time model in our formalization, however, the behavior of the kitchen timer is in fact sampling rate independent (Continuous Time FRP). Sampling rate independence is guaranteed in TSL as long as the next operator is not used. However, the relation between TSL with the next operator and Continuous Time FRP still needs to be explored.

In another direction, the usual way in FRP to distinguish between continuous and discrete behaviors is to use signals and events. So far we have embedded data into signals. It is open to future work how to utilize events natively. Future directions for improvements to usability include integrating FRP synthesis more tightly with programming, e.g., by allowing specifications to be used inline with QuasiQuoters [Mainland 2007].

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