Decision Procedures: Motivation

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Why Software Verification?

Maiden flight of the Ariane 5 rocket on the 4th of June 1996

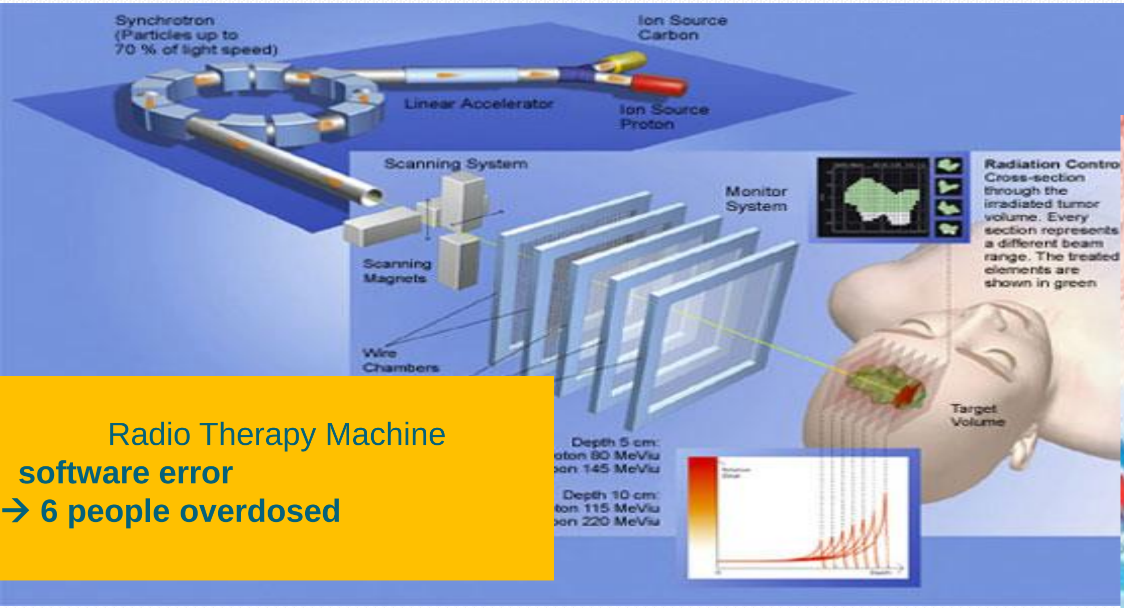
- The reason for the explosion was a software error
- Financial loss: $500,000,000 (including indirect costs: $2,000,000,000)

Air Transport

Boeing could not assemble and integrate the fly-by-wire system until it solved problems with the databus and the flight management software. Solving these problems took more than a year longer than Boeing anticipated. In April, 1995, the FAA certified the 777 as safe.

Total development cost:	$ 3 billion
Software integration and validation cost:	one third of total

Examples of Software Errors



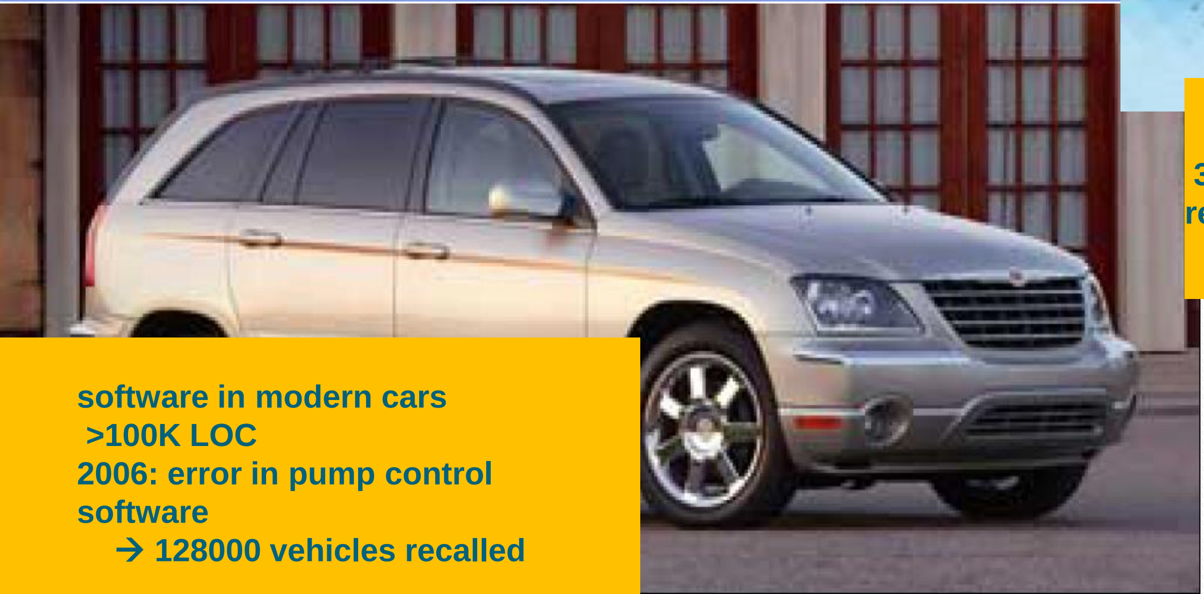
Radio Therapy Machine

software error

→ 6 people overdosed

Year 2010 Bug

30 million debit and credit cards have been rendered unreadable by the software bug



software in modern cars

>100K LOC

2006: error in pump control software

→ 128000 vehicles recalled

link

Financial Impact of Software Errors

Recent research at Cambridge University (2013, link) showed that the global cost of software bugs is

around 312 billion of dollars annually

Goal: to increase software reliability

How to obtain Software Reliability?

- Testing, testing, testing, …
  - Many software errors are detected this way
  - Does not provide any correctness guarantee
  - “Murphy’s Law”
- Verification
  - Provides a formal mathematical proof that a program is correct w.r.t. a certain property
  - A formally verified program will work correctly for every given input
  - Verification is algorithmically very hard task (problem is in general undecidable)

A Mathematical Proof of Program Correctness?

Can you verify my program?

```
public void add (Object x)

{
   Node e = new Node();
   e.data = x;
   e.next = root;
   root = e;
   size = size + 1;
}
```

Which property are you interested in?

Example Questions in Verification

- Will the program crash?
- Does it compute the correct result?
- Does it leak private information?
- How long does it take to run?
- How much power does it consume?
- Will it turn off automated cruise control?

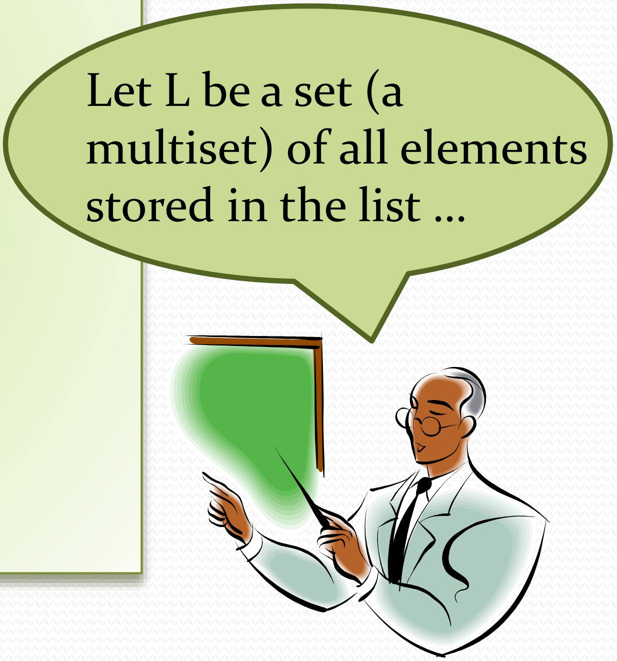
A Mathematical Proof of Program Correctness?

I just want to be sure that no element is lost in the list – if I insert an element, it is really there

```
    lic void add (Object x)



    ode e = new Node();
   e.data = x;
   e.next = root;
   root = e;
   size = size + 1;
}
```

A Mathematical Proof of Program Correctness?

```
//: L = data[root.next*]

public void add (Object x)

{
   Node e = new Node();
   e.data = x;
   e.next = root;
   root = e;
   size = size + 1;
}
```



Let L be a set (a multiset) of all elements stored in the list ...

A Mathematical Proof of Program Correctness?

Annotations

```
//: L = data[root.next*]
//: invariant: size = card L
public void add (Object x)
//: ensures L = old L + {x}
{
   Node e = new Node();
   e.data = x;
   e.next = root;
   root = e;
   size = size + 1;
}
```

Annotations

- Written by a programmer or a software analyst
- Added to the original program code to express properties that allow reasoning about the programs
- Examples:
  - Preconditions:
    - Describe properties of an input
  - Postconditions:
    - Describe what the program is supposed to do
  - Invariants:
    - Describe properties that have to hold in every program point

Decision Procedures for Collections

```
//: L = data[root.next*]
//: invariant: size = card L
public void add (Object x)
//: ensures L = old L + {x}
{
   Node e = new Node();
   e.data = x;
   e.next = root;
   root = e;
   si
}
```

Prove that the following formula always holds:

$$\forall X.\ \forall L.\ |X| = 1 \rightarrow |\, L \uplus X \,| = |L| + 1$$

Verification condition

Verification Conditions

- Mathematical formulas derived based on:
  - Code
  - Annotations
- If a verification condition always holds (valid), then to code is correct w.r.t. the given property
- It does not depend on the input variables
- If a verification condition does not hold, we should be able to detect an error in the code

Verification Condition: Example

```
//: assume (x > 0)
def simple (Int x)
//: ensures y > 0
{
  ??
  return y
}
```

Verification Condition: Example

```
//: assume (x > 0)
def simple (Int x)
//: ensures y > 0
{
  val y = x - 2
  return y
}
```

Verification condition:

$$\forall x.\ \forall y.\ x > 0 \ \wedge y = x - 2 \rightarrow y > 0$$

Verification Condition: Example

```
//: assume (x > 0)
def simple (Int x)
//: ensures y > 0
{
  val y = x - 2
  return y
}
```

Verification condition:

$$\forall x. \forall y. x > 0 \wedge y = x - 2 \rightarrow y > 0$$

Preconditions

Verification Condition: Example

```
//: assume (x > 0)
def simple (Int x)
//: ensures y > 0
{
  val y = x - 2
  return y
}
```

Verification condition:

$$\forall x.\ \forall y.\ x > 0 \ \wedge y = x - 2 \rightarrow y > 0$$

Program

Verification Condition: Example

```
//: assume (x > 0)
def simple (Int x)
//: ensures y > 0
{
  val y = x - 2
  return y
}
```

Verification condition:

$$\forall x. \forall y. x > 0 \wedge y = x - 2 \rightarrow y > 0$$

Postconditions

Verification Condition: Example

```
//: assume (x > 0)
def simple (Int x)
//: ensures y > 0
{
  val y = x - 2
  return y
}
```

Verification condition:

$$\forall x.\ \forall y.\ x > 0 \ \wedge y = x - 2 \rightarrow y > 0$$

Formula does not hold for input x = 1

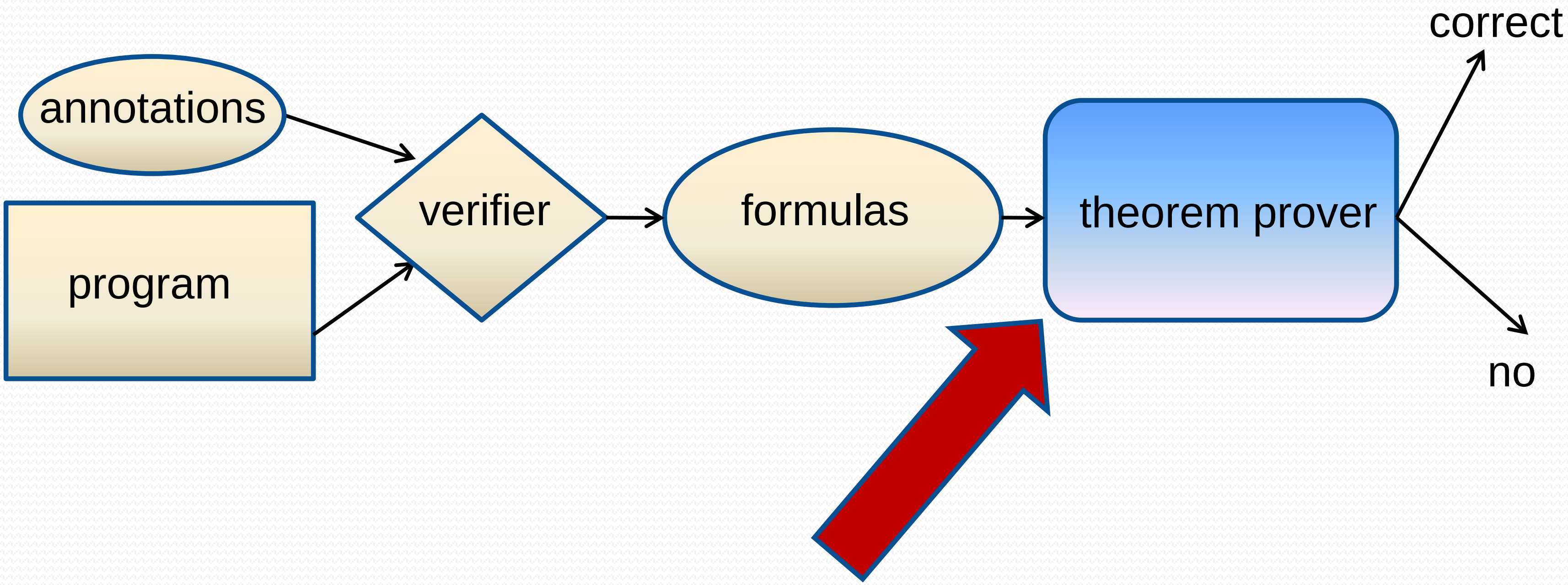
Automation of Verification

- Windows XP has approximately 45 millions lines of source code

$\cong$ 300.000 DIN A4 papers
$\cong$ 12m high paper stack

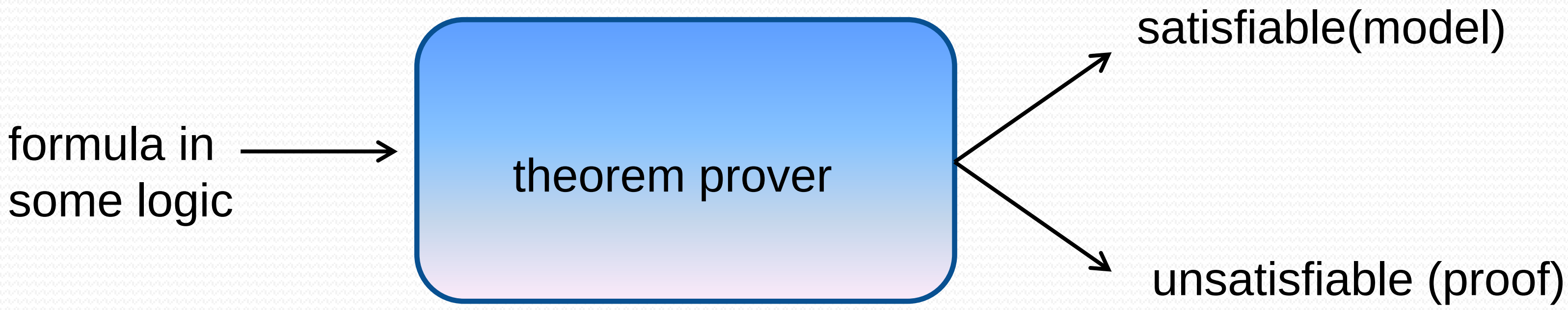
Verification should be automated!!!

Software Verification



Prove formulas automatically!

Decision Procedures



formula in
some logic

theorem prover

satisfiable(model)

unsatisfiable (proof)

- A decision procedure is an algorithm which answers whether the input formula is satisfiable or not
    - formula $x \leq y$ is satisfiable for x=0, y=1
    - formula $x \leq y \wedge x+1 > y+1$ is unsatisfiable

Combining Various Logics

```
//: L = data[root.next*]
//: invariant: size = card L
public void add (Object x)
//: ensures L = old L + {x}
{
  Node e = new Node();
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Combining Various Logics

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}
```

Verification condition:

$$
\begin{aligned}
&\neg\, next_0{}^*(root_0,n) \wedge x \notin \{data_0(v)\ |next_0{}^*(root_0,v)\}\\
&\wedge\, next=next_0\,[n:=root_0] \wedge data=data_0[n:=x] \rightarrow\\
&|\{data(v)\ .\ next^*(n,v)\}| =\\
&\qquad\qquad |\{data_0(v)\ .\ next_0{}^*(root_0,v)\}| + 1
\end{aligned}
$$

Another Application of Decision Procedures: Software Synthesis

- Software synthesis = a technique for automatically generating code given a specification
- Why?
  - ease software development
  - increase programmer productivity
  - fewer bugs
- Challenges
  - synthesis is often a computationally hard task
  - new algorithms are needed

Software Synthesis

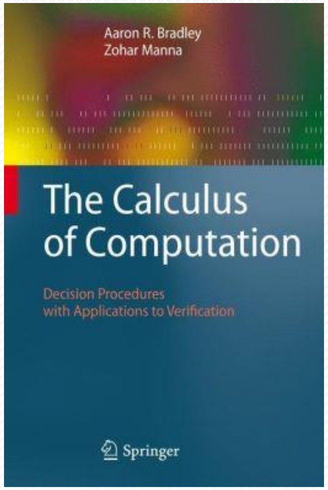
```
val bigSet = ....


val (setA, setB) = choose((a: Set, b: Set) ) =>
   ( a.size == b.size  && a union b == bigSet && a intersect b == empty))
```

Code

```
assert (bigSet.size % 2  == 0)
val n = bigSet.size/2
val setA = take(n, bigSet)
val setB = bigSet –– setA
```

Course Textbooks



Aaron R. Bradley
Zohar Manna

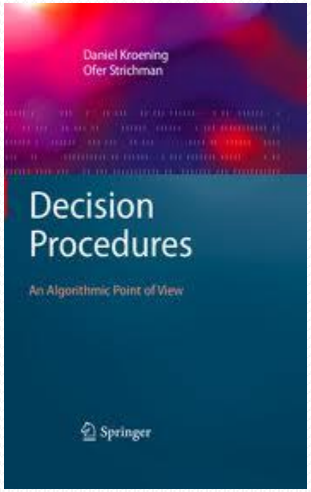
The Calculus of Computation

Decision Procedures with Applications to Verification

Springer

Aaron R. Bradley, Zohar Manna: *The calculus of computation - decision procedures with applications to verification.* Springer 2007

Daniel Kroening, Ofer Strichman: *Decision Procedures: An Algorithmic Point of View.* Springer 2008



Daniel Kroening
Ofer Strichman

Decision Procedures

An Algorithmic Point of View

Springer