On Optimal Diversity in Network-Coding-Based Routing in Wireless Networks

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Outline

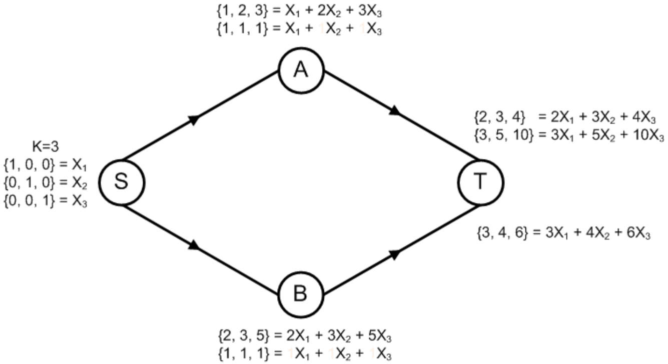
1. Introduction
2. System Model and Problem Definition
3. An Analytical Framework for NC-based Routing
4. Optimizing Diversity of NC-based Routing
5. ONCR: an Optimal NC-Based Routing Protocol
6. Performance Evaluation
7. Conclusion and Future Work

Introduction

Network Coding (NC)

- First proposed in wired networks
- Provide benefits on throughput and robustness
- Naturally extended into wireless environment
- Network-Coding-based routing, e.g., MORE, CodeOR, CCACK and MIXIT

Network-Coding-Based Routing: An Example



Differences from Opportunistic Routing

- Packets are divided into batches and encoded
- No communication needed between forwarders
  - Every node broadcasts

Network-Coding-Based Routing Protocols

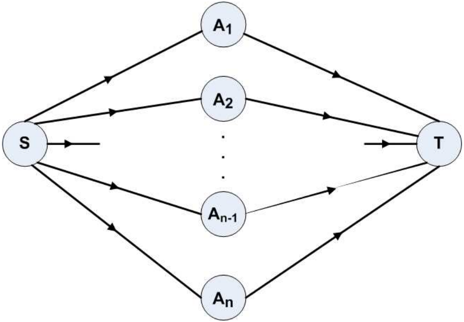
Three key challenges:

- How to select the set of forwarders? **Routing Diversity Control**
- When to stop broadcasting? **ACK Scheme**
- When to start broadcasting? **Rate Control**

Existing protocols:

- Focus on improving network throughput
- Various ACK and rate control schemes are proposed
- **Routing diversity control is overlooked**

Routing Diversity



Utilizing all routing diversity

- High contention and collision
- Compromising network throughput and data delivery cost
- Not suitable for resource-constrained wireless networks, e.g., sensor networks

Our Contribution

- An analytical framework for estimating the cost of NC-based routing
- A greedy optimal algorithm to minimize the cost of NC-based routing
- ONCR, a fully distributed minimal cost NC-based routing protocol
- Performance improvement of ONCR over state-of-the-art protocols on sensor testbed

Outline

System Model and Problem Definition

System model

- A directed graph $G = (V, E)$ with one source S and one destination T
- Edge $(i, j) \in E$ with link reliability $P_{ij} = \frac{1}{ETX_{ij}}$
- Node i has a forwarder candidate set FCS_i, i.e., one-hop neighbors of i

System Model and Problem Definition

EST-NC Problem

- Let forwarder set $FS_i = FCS_i$ for each node i
- **Estimate** the total transmission cost to deliver K linear independent packets from S to T.

MIN-NC Problem

- Determine the forwarder set FS_i for each node i
- **Minimize** the total transmission cost to deliver K linear independent packets from S to T

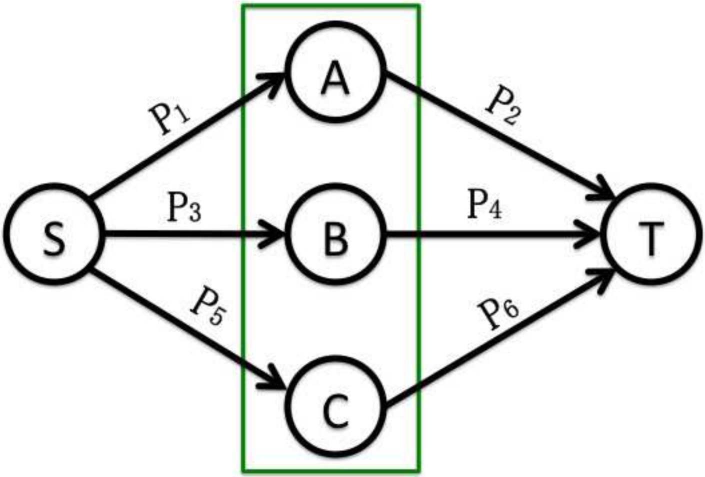
Outline

An Analytical Framework for NC-based Routing

How does it work?

1. Define the whole forwarder set as a **virtual node** V_S
2. Compute the transmission cost from the S to V_S
3. Sort forwarders in non-descending order of their transmission cost
4. Each forwarder only forwards its **effective load** with corresponding cost
5. Sum up all transmission cost

An example



$$C_{SV_S}(K) = \frac{K}{P_{SV_S}} = \frac{K}{1-(1-P_1)(1-P_3)(1-P_5)}$$

An Analytical Framework for NC-based Routing

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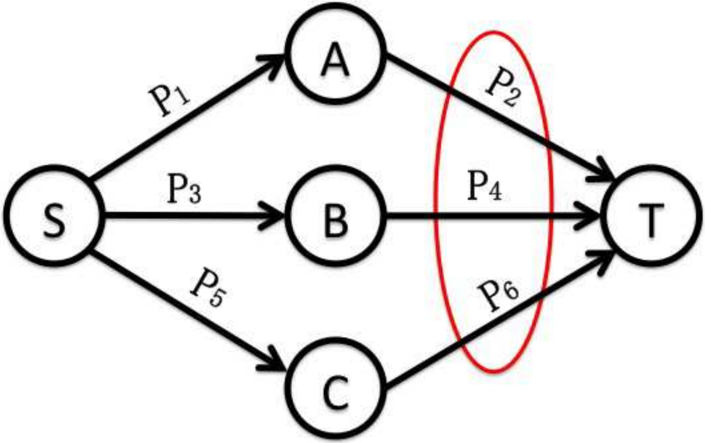
An example



$$P_2 \geq P_4 \geq P_6$$

An Analytical Framework for NC-based Routing

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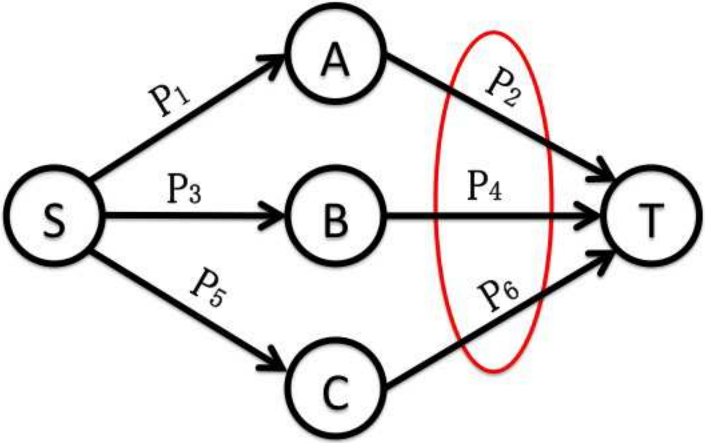
An Analytical Framework for NC-based Routing

Definition

For a node j in the forwarder candidate set FCS_i, the **effective load** L_j is defined as the number of linear independent packets received by j but none of the nodes in FCS_i that has lower transmission cost to the destination.

An example



$$P_2 \geq P_4 \geq P_6$$

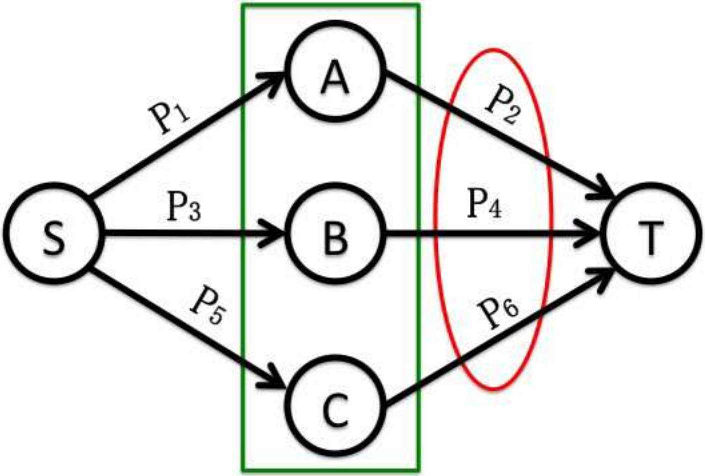
Packets Recevied

$$K_A^S = \frac{KP_1}{1-(1-P_1)(1-P_3)(1-P_5)}$$
$$K_B^S = \frac{KP_3}{1-(1-P_1)(1-P_3)(1-P_5)}$$
$$K_C^S = \frac{KP_5}{1-(1-P_1)(1-P_3)(1-P_5)}$$

Effective Load

$$L_A = K_A^S$$
$$L_B = K_B^{S'} = K\frac{K_B^S}{K}(1-P_1) = K_B^S(1-P_1)$$
$$L_C = K_C^{S'} = K_C^S(1-P_1)(1-P_3)$$

An example



$$
\begin{aligned}
C_S(K) &= C_{SD_S}(K) + C_{AT}(L_A) + C_{BT}(L_B) + C_{CT}(L_C) \\
&= \frac{K}{1-(1-P_1)(1-P_3)(1-P_5)} \\
&\quad + \frac{L_A}{P_2} + \frac{L_B}{P_4} + \frac{L_C}{P_6} \\
&= \frac{K}{1-(1-P_1)(1-P_3)(1-P_5)} \\
&\quad \cdot [1 + \frac{P_1}{P_2} + \frac{P_3(1-P_1)}{P_4} + \frac{P_5(1-P_1)(1-P_3)}{P_6}]
\end{aligned}
$$

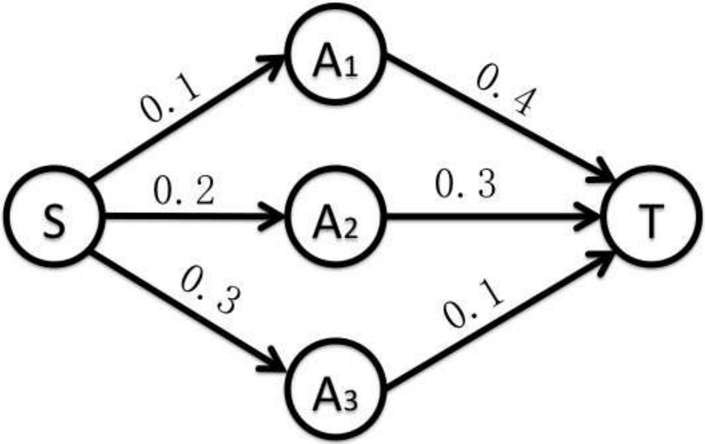
Outline

Optimizing Diversity of NC-based Routing

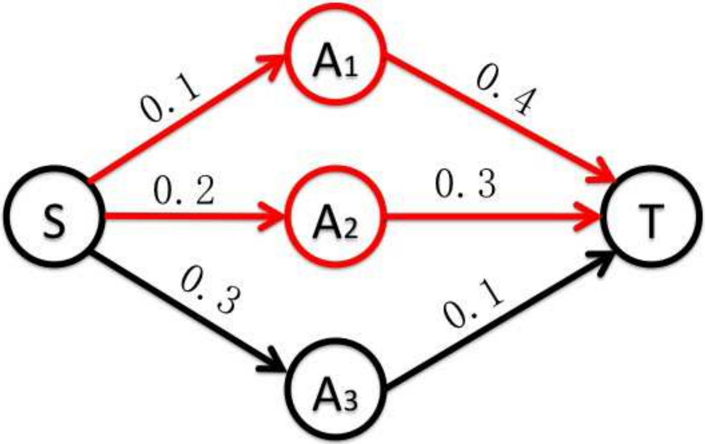
A greedy algorithm

1. Sort forwarder candidates in non-descending order of their transmission cost;
2. Select the best candidate remaining into forwarder set;
3. Keep it in the set if the total transmission cost can be reduced, go back to last step;
4. Stop if the total transmission cost cannot be reduced.

Optimizing Diversity of NC-based Routing



Optimizing Diversity of NC-based Routing



The optimal forwarder set is $\{A_1, A_2\}$

$$
\begin{aligned}
C_{\{A_1,A_2\}}(K) &= \frac{K}{1-(1-0.1)(1-0.2)} \cdot [1 + \frac{0.1}{0.4} + \frac{0.2(1-0.1)}{0.3}] \\
&= \frac{K}{0.28} \cdot (1 + \frac{1}{4} + \frac{0.18}{0.3}) \\
&= 6.6071K \\
C_{\{A_1,A_2,A_3\}}(K) &= \frac{K}{1-(1-0.1)(1-0.2)(1-0.3)} \\
&\quad \cdot [1 + \frac{0.1}{0.4} + \frac{0.2(1-0.1)}{0.3} + \frac{0.3(1-0.1)(1-0.2)}{0.1}] \\
&= \frac{K}{0.496} \cdot (1 + \frac{1}{4} + \frac{0.18}{0.3} + \frac{0.216}{0.1}) \\
&= 8.0847K > C_{\{A_1,A_2\}}(K)
\end{aligned}
$$

Theorem of Optimality

Theorem

Given a node S and its forwarder candidate set $D_S = \{A_1, A_2, \ldots, A_M\}$, the proposed greedy algorithm yields the minimal transmission cost to the destination node of NC-based routing and the corresponding forwarder set.

We proved this theorem by contradiction.

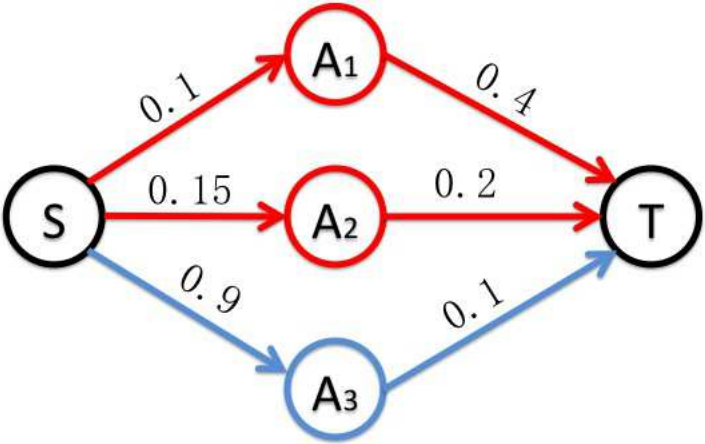
Properties of Optimal Diversity

Theorem

Given a node S with a candidate set FCS_S of M forwarders, the optimal forwarder set FS_S computed in the proposed greedy algorithm does not always contain node A^* where $A^* \in FCS_S$ and $\frac{K}{P_{SA^*}} + C_{A^*}(K) \leq \frac{K}{P_{SA_i}} + C_{A_i}(K)$ for any $i \in FCS_S/\{A^*\}$.

Shortest single path routing is not always in the optimal routing diversity.

Properties of Optimal Diversity



The optimal forwarder set is $\{A_1, A_2\}$

$$
\begin{aligned}
C_{\{A_1,A_2\}}(K) &= \frac{K}{1-(1-0.1)(1-0.15)} \cdot [1 + \frac{0.1}{0.4} + \frac{0.15(1-0.1)}{0.2}] \\
&= \frac{K}{0.235} \cdot (1 + \frac{1}{4} + \frac{0.135}{0.2}) \\
&= 8.1915K \\
C_{\{A_1,A_2,A_3\}}(K) &= \frac{K}{1-(1-0.1)(1-0.15)(1-0.9)} \\
&\quad \cdot [1 + \frac{0.1}{0.4} + \frac{0.15(1-0.1)}{0.2} + \frac{0.9(1-0.1)(1-0.15)}{0.1} \\
&= \frac{K}{0.9235} \cdot (1 + \frac{1}{4} + \frac{0.135}{0.2} + \frac{0.6885}{0.1}) \\
&= 9.5398K > C_{\{A_1,A_2\}}(K)
\end{aligned}
$$

Properties of Optimal Diversity

Theorem

Given a node S *with a candidate set* FCS_S *of* M *forwarders, the optimal transmission cost* $C_S^*(K)$ *computed in the proposed greedy algorithm is always lower than or equal to* $\frac{K}{P_{SA^*}} + C_{A^*}(K)$ *where* $A^* \in FCS_S$ *and* $\frac{K}{P_{SA^*}} + C_{A^*}(K) \leq \frac{K}{P_{SA_i}} + C_{A_i}(K)$ *for any* $i \in FCS_S / \{A^*\}$.

Cost of optimal NC-based routing is upper bounded by shortest single path routing.

Outline

ONCR: an Optimal NC-Based Routing Protocol

- **Routing Engine**: a distributed implementation of the proposed greedy algorithm
- **M-NSB**: a coded ACK scheme to solve the collective space problem with lower implementation complexity than CCACK
- **Rate Control**: nodes forward a flow after receiving a load-dependent threshold of packets to 1) reduce contention and 2) avoid potential linear dependence between forwarded packets

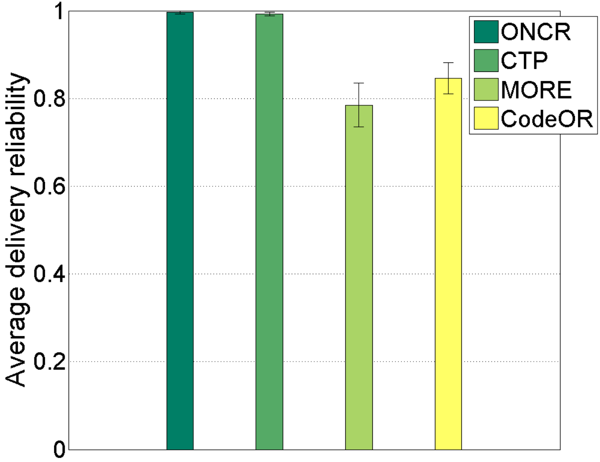
Outline

Performance Evaluation

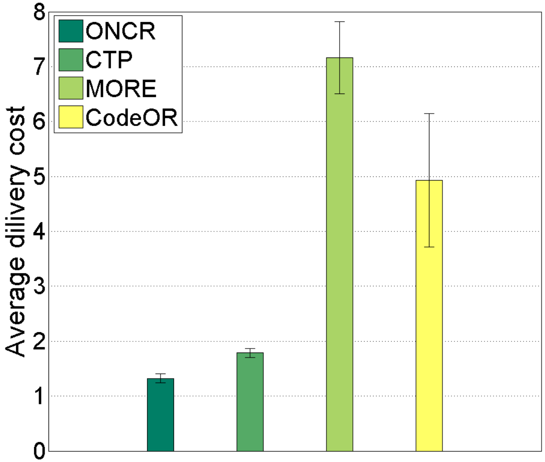
Experiment setting up

- Testbed: NetEye, a 130-sensor testbed at Wayne State University
- Topology: 40 nodes, 10/20 are source nodes, 1 sink node
- Protocols compared: *ONCR*, CTP, MORE, CodeOR
- Traffic pattern: 3-second periodic traffic
- Metrics: delivery reliability, delivery cost, goodput and routing diversity

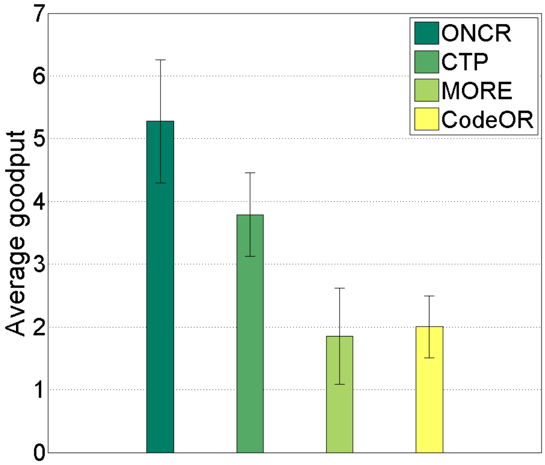
10-source: delivery reliability



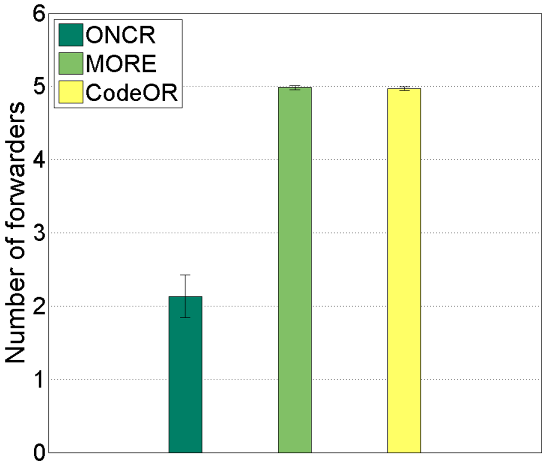
10-source: delivery cost



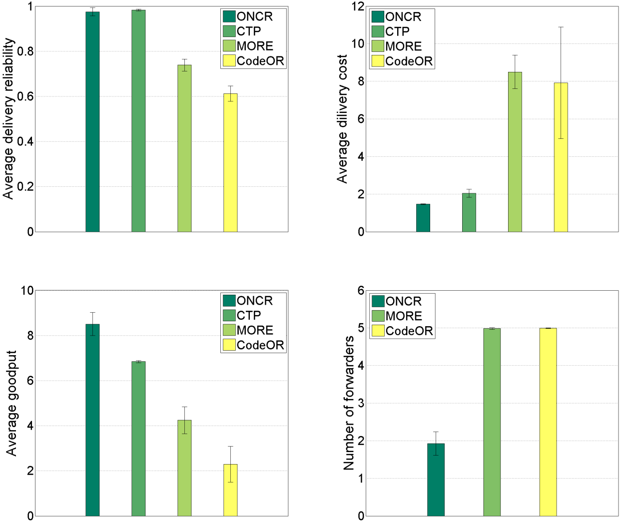
10-source: goodput



10-source: routing diversity



20-source


Average delivery reliability
ONCR
CTP
MORE
CodeOR
Average dilivery cost
ONCR
CTP
MORE
CodeOR
Average goodput
ONCR
CTP
MORE
CodeOR
Number of forwarders
ONCR
MORE
CodeOR

Outline

Conclusion and Future Work

Conclusion

- Analytical framework on cost of NC-based routing
- A greedy minimal cost algorithm for NC-based routing
- ONCR: an optimal NC-based routing protocol
- Experimental evaluation on sensor network testbed

Future Work

- How to utilize the remaining routing diversity?
    - NC-based protection

Algorithm 1 Compute the transmission cost of NC-based routing for the current node S with M forwarder candidates

```
Input: current node S, D_S = {A_1, A_2, ..., A_M}
Output: C_S(1): the expected number of transmissions to deliver 1 packet from S to T
Sort nodes in D_S by a non-descending order of C_{A_i}(1), where i = 1, 2, ..., M.
Sorted nodes are labeled as {A'_1, A'_2, ..., A'_M}
C_{SD_S}(1) = 1 / (1 - ∏_{i=1}^{M} (1 - P_{SA'_i}))
L_{A'_1} = C_{SD_S}(1) P_{SA'_1}
F = 1 - P_{SA'_1}
for i → 2, 3, ..., M do
    L_{A'_i} = C_{SD_S}(1) P_{SA'_i} F
    C_{A'_i}(L_{A'_i}) = L_{A'_i} C_{A'_i}(1)
    F = F(1 - P_{SA'_i})
end for
C_S(1) = C_{SD_S}(1) + Σ_{i=1}^{M} C_{A'_i}(L_{A'_i})
```

In LaTeX notation:

Input: current node S, $D_S = \{A_1, A_2, \ldots, A_M\}$

Output: $C_S(1)$: the expected number of transmissions to deliver 1 packet from S to T

Sort nodes in D_S by a non-descending order of $C_{A_i}(1)$, where $i = 1, 2, \ldots, M$.

Sorted nodes are labeled as $\{A'_1, A'_2, \ldots, A'_M\}$

$C_{SD_S}(1) = \frac{1}{1-\prod_{i=1}^{M}(1-P_{SA'_i})}$

$L_{A'_1} = C_{SD_S}(1)P_{SA'_1}$

$F = 1 - P_{SA'_1}$

for $i \to 2, 3, \ldots, M$ **do**

  $L_{A'_i} = C_{SD_S}(1)P_{SA'_i}F$

  $C_{A'_i}(L_{A'_i}) = L_{A'_i}C_{A'_i}(1)$

  $F = F(1 - P_{SA'_i})$

end for

$C_S(1) = C_{SD_S}(1) + \sum_{i=1}^{M} C_{A'_i}(L_{A'_i})$

Algorithm 2 Compute the minimal transmission cost of NC-based routing and the corresponding FS for the input node S with M forwarders

Input: node S, $D_S = \{A_1, A_2, \ldots, A_M\}$, $FS_S = \emptyset$
Output: $C_S^*(1)$: the minimal transmission cost to deliver 1 packet from S to T
Sort nodes in D_S by a non-descending order of $C_{A_i}(1)$, where $i = 1, 2, \ldots, M$.
Sorted nodes are labeled as $\{A'_1, A'_2, \ldots, A'_M\}$
$FS_S = \{A'_1\}$
$C_S^*(1) = \frac{1}{P_{SA'_1}} + C_{A'_1}(1)$
for $i \rightarrow 2, 3, \ldots, M$ **do**
    Run Algorithm 1 with input S and $D_S = \{A'_1, \ldots, A'_i\}$
    Get the result as $C_S^{new}(1)$
    if $C_S^{new}(1) > C_S(1)$ **then**
        break
    else
        $FS_S = FS_S \cup A'_i$
        $C_S^*(1) = C_S^{new}(1)$
    end if
end for