This is just to test your background

I wrote these problems to help you decide if you know enough discrete math to take this course. You should be able to solve these problems after a little thought. I am not yet sure if I will write solutions to these.

Problem 1: Defining a Tree

A graph $T$ with $n$ vertices is called a tree if it satisfies any two of the following three properties:

1. $T$ has no cycles.
2. $T$ has $n-1$ edges.
3. $T$ is connected.

Prove that any of those two imply the third.

Problem 2: Re-defining a Tree

Here’s another definition of a tree: a tree is a connected graph such that for every two vertices $x$ and $y$, there is exactly one path in the graph between $x$ and $y$. Prove that this definition is equivalent to the definition from Problem 1.

Problem 3: Another property of trees.

Let $T$ be a tree. Prove that $T$ contains a $v$ vertex of degree 1. Let $(u, v)$ be the unique edge involving $v$. Prove that if we remove $v$ and edge $(u, v)$ from $T$, then the graph that remains is also a tree.