Multiplicative Weights Method note

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4:00 PM

Here is the analysis implied at the end of Theorem 1:

We have

$$(1-\varepsilon)^{m_i^t} \le n\left(1-\frac{\varepsilon}{2}\right)^{m^t}$$

by $(1-x) \le e^{-x}$, for $x \ge 0$, we get

$$(1-\varepsilon)^{m_i^t} \le n e^{-\frac{\varepsilon}{2} m^t}$$

Taking logs, this gives

$$m_i^t \log(1-\varepsilon) \le \log(n) - \frac{\varepsilon}{2} m^t$$

$$\Longleftrightarrow \quad \frac{\varepsilon}{2} \cdot m^t \le \log(n) + m_i^t\left(-\log(1-\varepsilon)\right)$$

so $$m^t \le \frac{2\log(n)}{\varepsilon} + m_i^t\left(\frac{2}{\varepsilon}\left(-\lg(1-\varepsilon)\right)\right)$$

applying $-\lg(1-\varepsilon) \le \varepsilon + \varepsilon^2$, for $\varepsilon \le \frac{1}{2}$,

we get

$$\frac{2}{\varepsilon}\left(-\lg(1-\varepsilon)\right) \le 2(1+\varepsilon), \text{ so}$$

$$m^t \le \frac{2\log(n)}{\varepsilon} + 2(1+\varepsilon)M_i^t$$