Lecture 4 on NP

Today

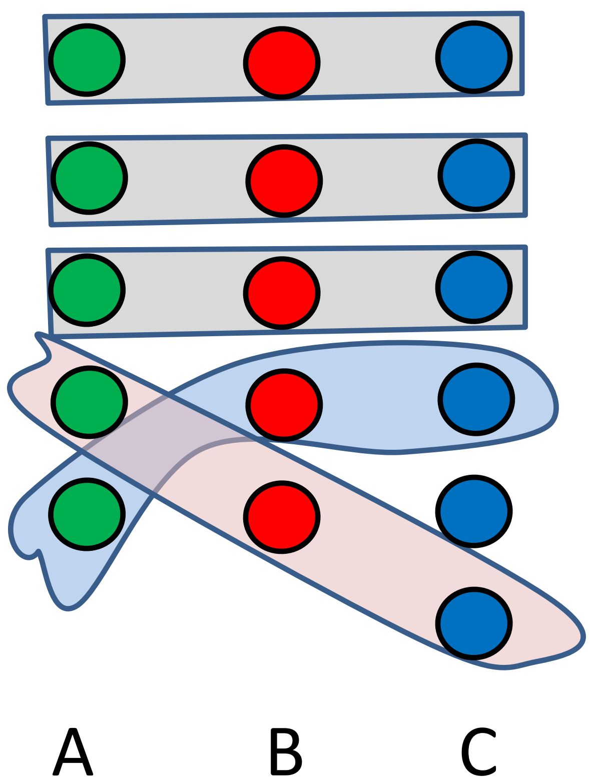
Will prove more problems are NP-complete:

3D Matching
Generalized 3DM
Exact Cover
Subset Sum
Interval Sched with Deadlines
and Release Times (ISDR)

Generalized 3DM

Given three sets, A, B, C, $|A| = |B| = k$
and triples $T_1, \ldots, T_n$, each with one element of A, B, and C

Do there exist k pairwise disjoint triplets?



Equivalent: disjoint triplets that cover all of A and B.

Gen-3DM is NP-Complete

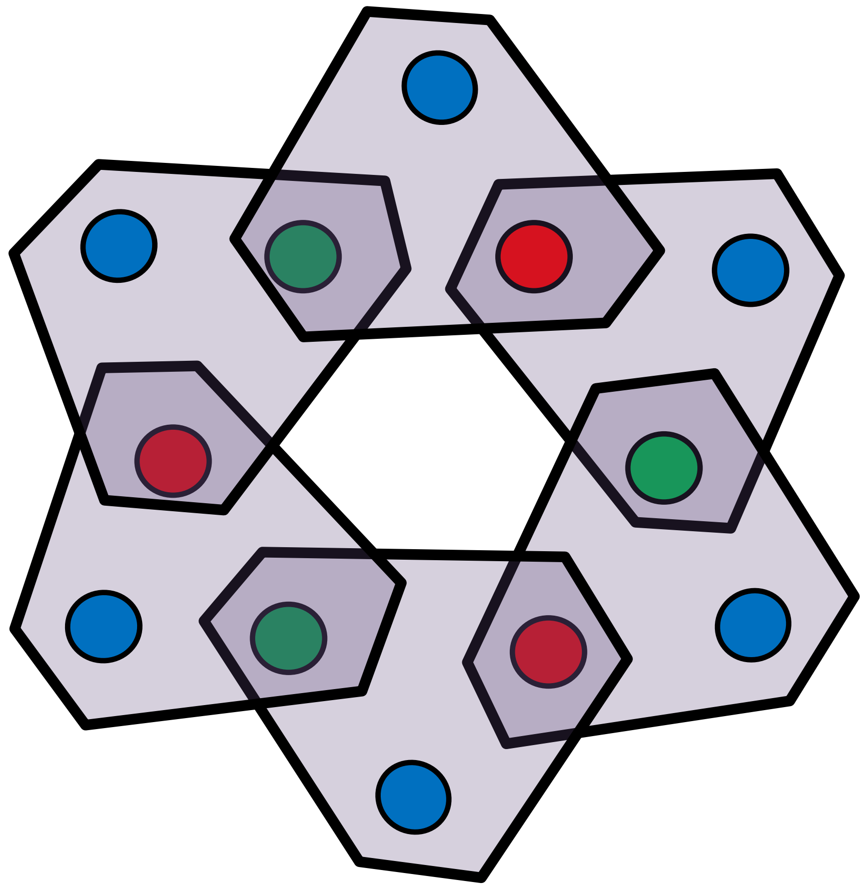
Clearly in NP, because can check a proposed matching.
To prove NP-hard, will show 3-SAT $\leq_P$ Gen-3DM.

Given an collection of clauses $C_1, \ldots, C_k$, each with at most 3 terms, on variables $x_1, \ldots, x_n$

produce sets A, B, C, and triples $S_1, \ldots, S_m$
that have matching iff the clauses are all satisfiable

Gen-3DM NP-Complete – variable gadgets

If these are only triples containing inner elements, must cover by all odd or all even triples



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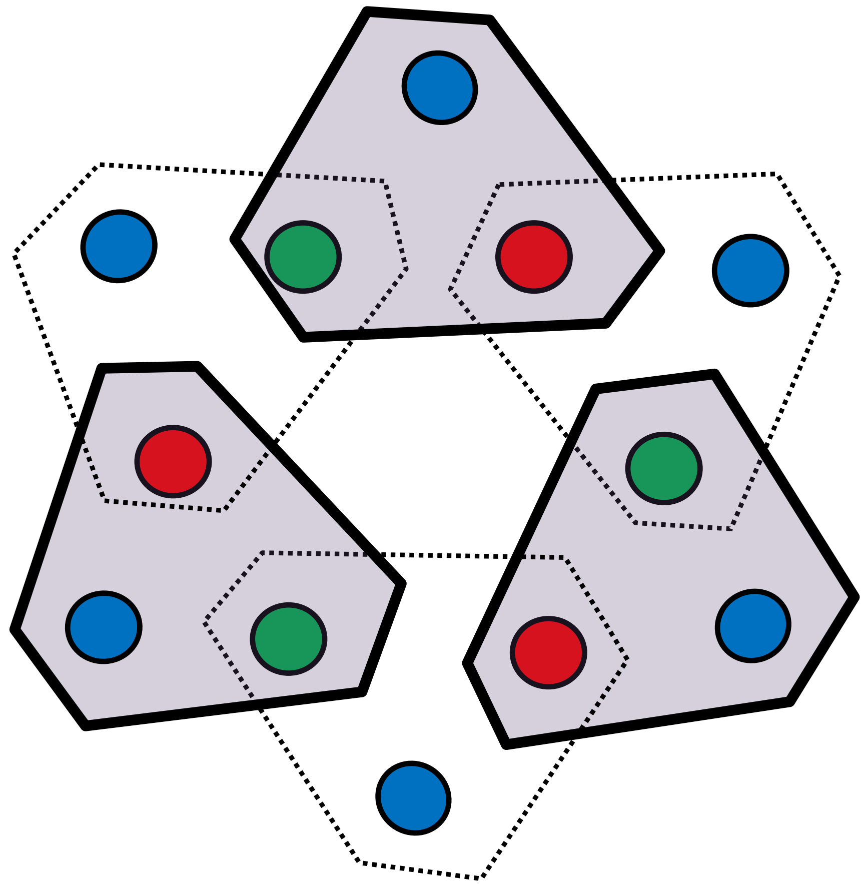

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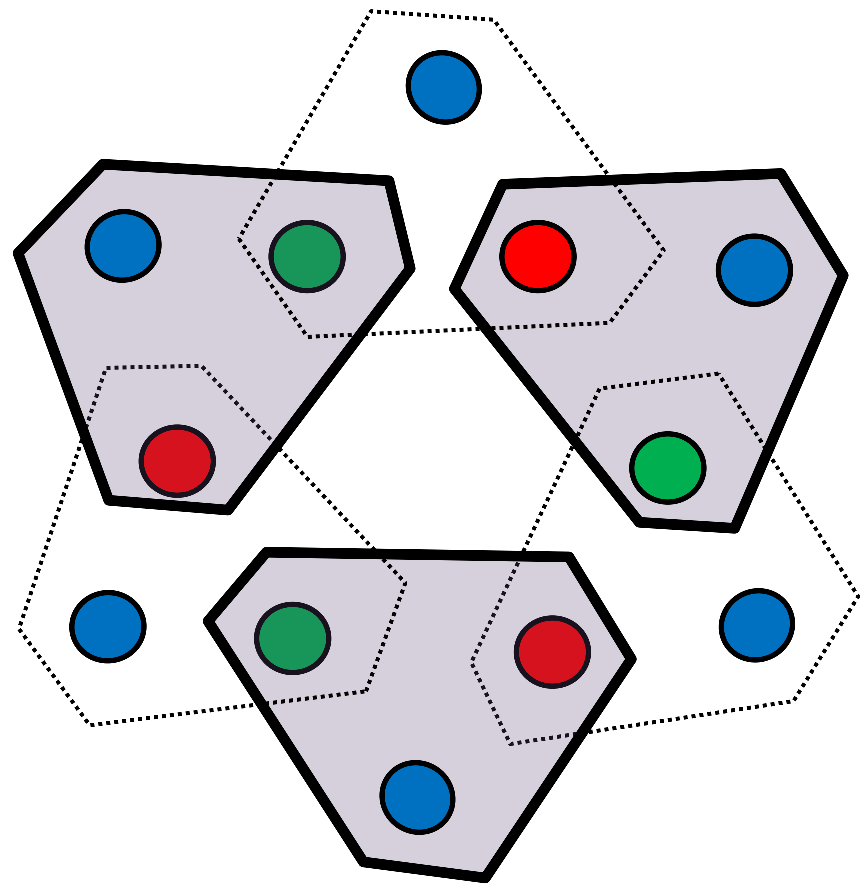


3DM NP-Complete – variable gadgets

For variable x_i in d clauses, create gadget with 2d inner elements:

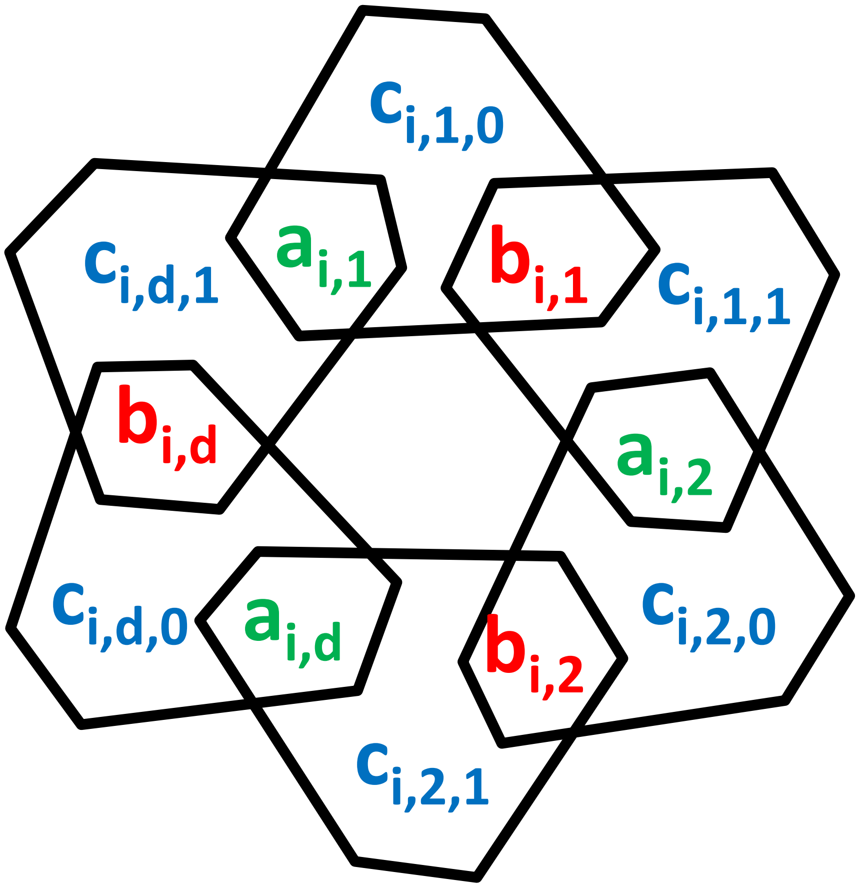
$a_{i,1}, a_{i,2}, \ldots, a_{i,d}$

$b_{i,1}, b_{i,2}, \ldots, b_{i,d}$

and 2d outer elements

$c_{i,1,0}, c_{i,2,0}, \ldots, c_{i,d,0},$

$c_{i,1,1}, c_{i,2,1}, \ldots, c_{i,d,1}$



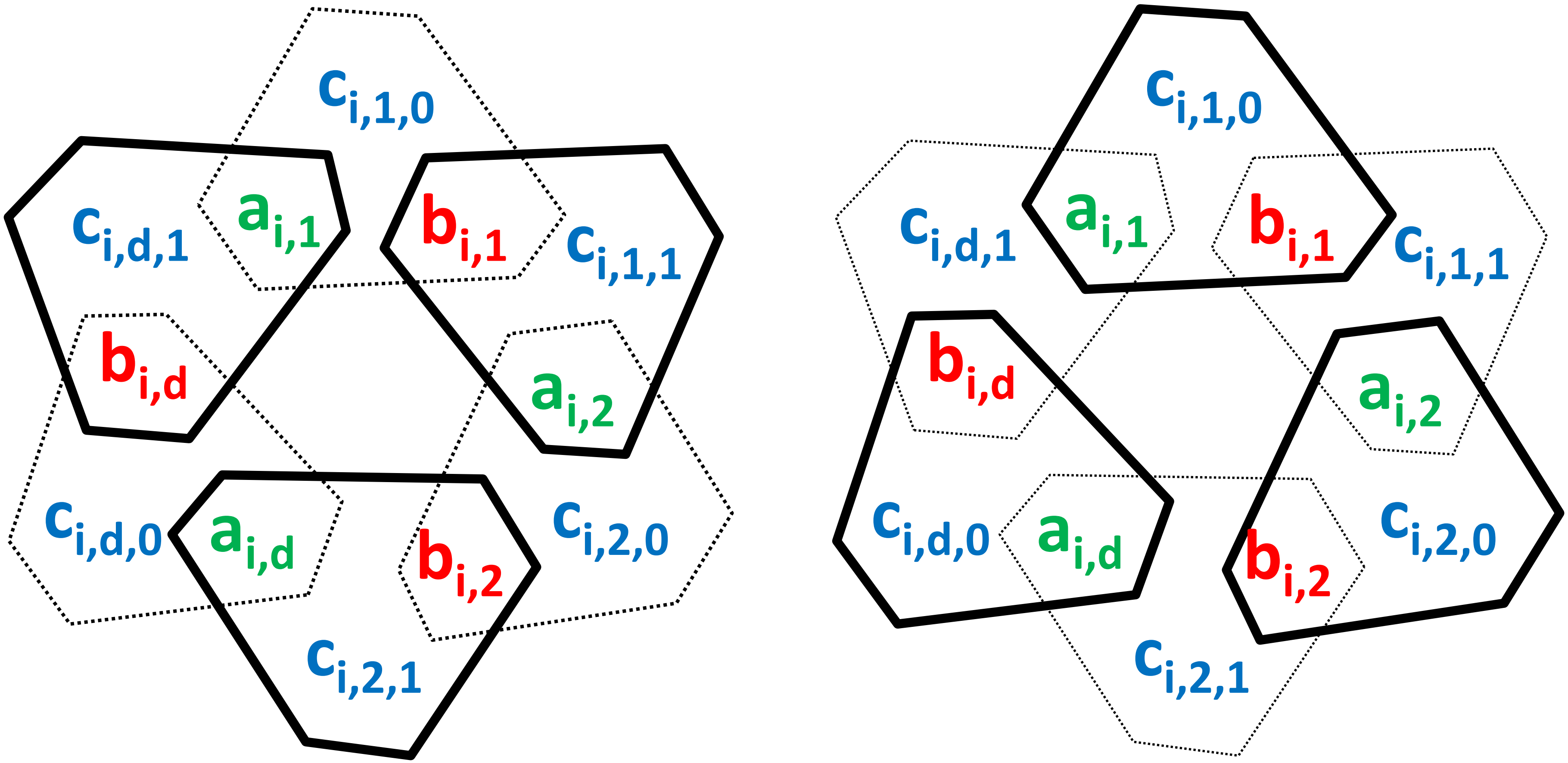
and triples as shown:

$(a_{i,k}, b_{i,k}, c_{i,k,0}), (a_{i,k+1}, b_{i,k}, c_{i,k,1})$

3DM NP-Complete – variable gadgets

Interpret covering inner elements by odd sets as false.

Interpret covering inner elements by even sets as true



Expose $c_{i,*,0}$

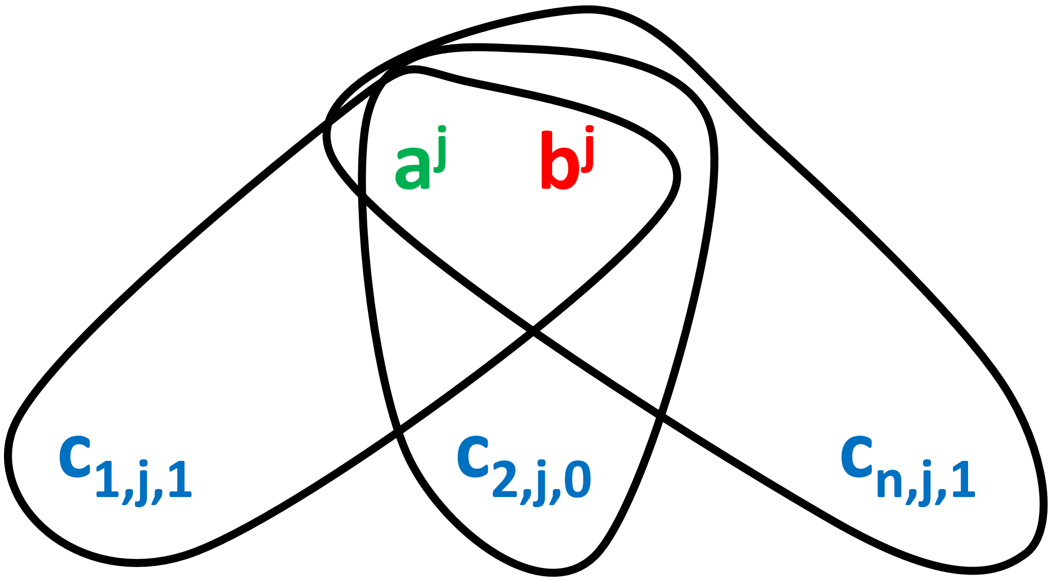
Expose $c_{i,*,1}$

3DM NP-Complete – clause gadget

Say clause C^j has form $x_1 \vee \overline{x_2} \vee x_n$

Create two elements for the clause:

a^j and b^j

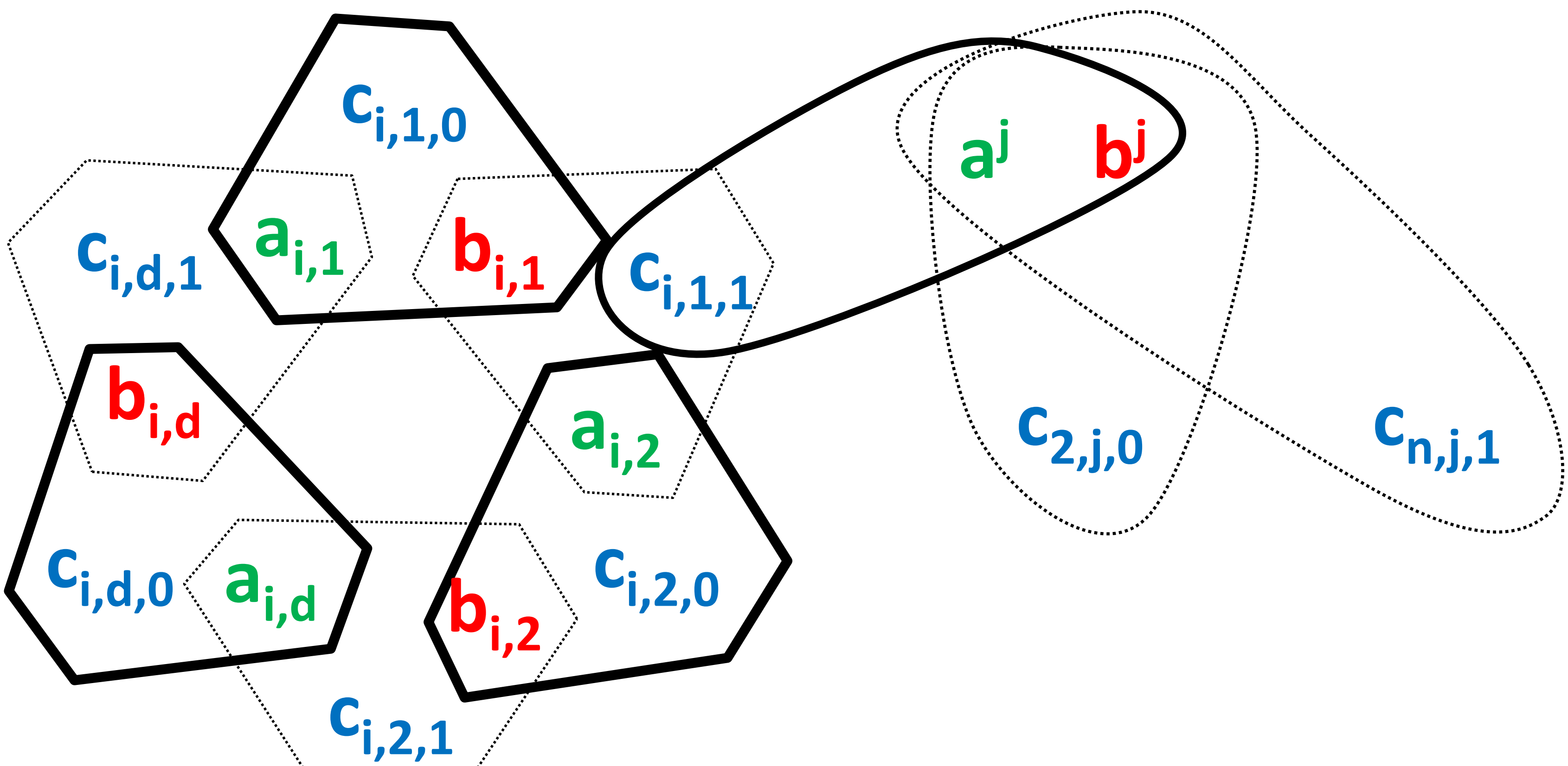


and create triples with these and terms that satisfy clause : $(a^j, b^j, c_{1,j,1})$, $(a^j, b^j, c_{2,j,0})$, $(a^j, b^j, c_{n,j,1})$,

3DM NP-Complete – clause gadget

Say clause Cj has form $x_1 \vee \overline{x_2} \vee x_n$

If these are only triples with the clause elements, must cover by a variable's external element that satisfies clause, and variable gadgets enforce consistency



3DM NP-Complete – clause gadget

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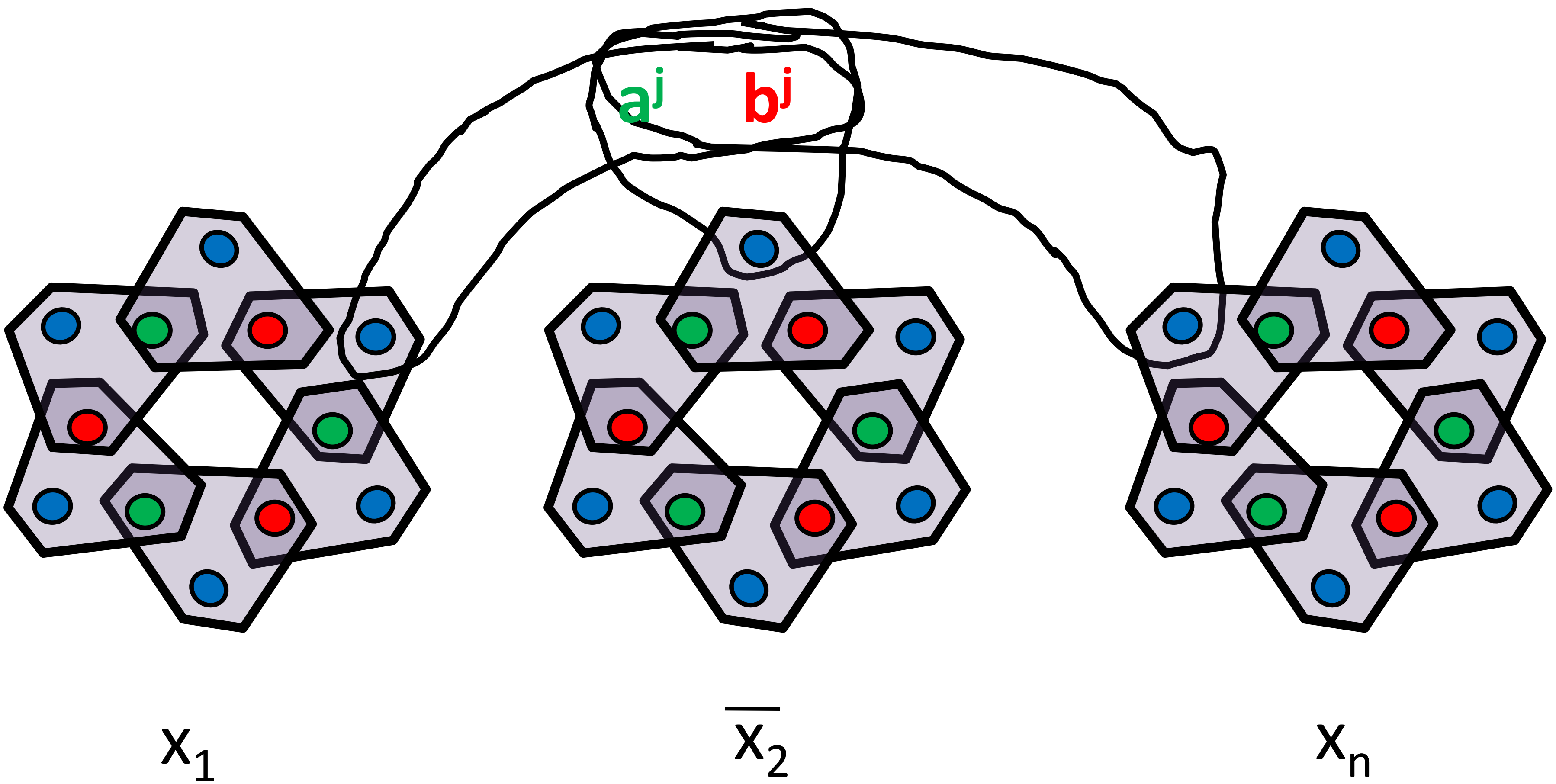
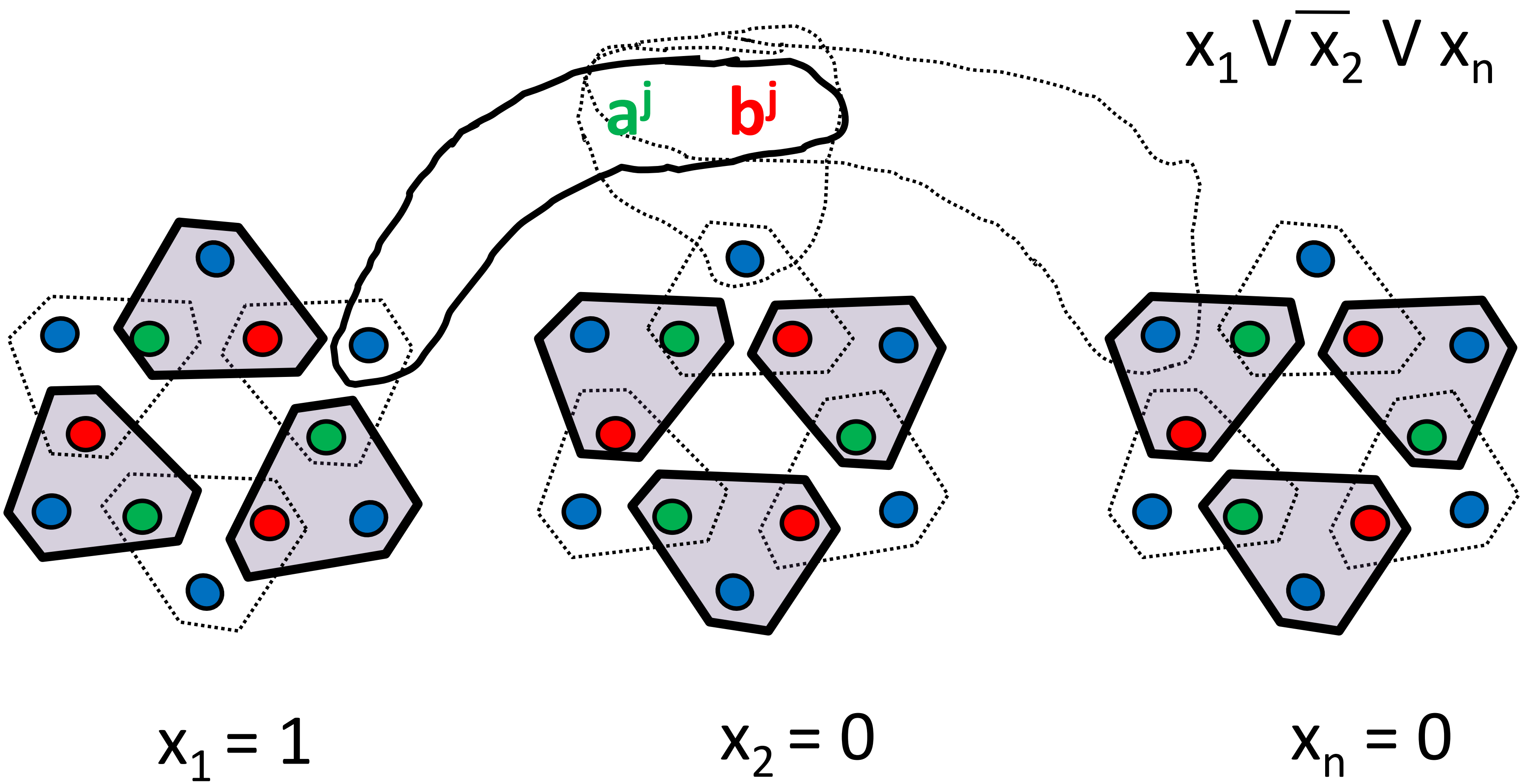


Each clause gets own external element for each variable

Truth assignment -> choice of triples at variable gadgets.
Satisfying -> can choose a triple for each clause gadget.

Disjoint, and cover all of A and B.



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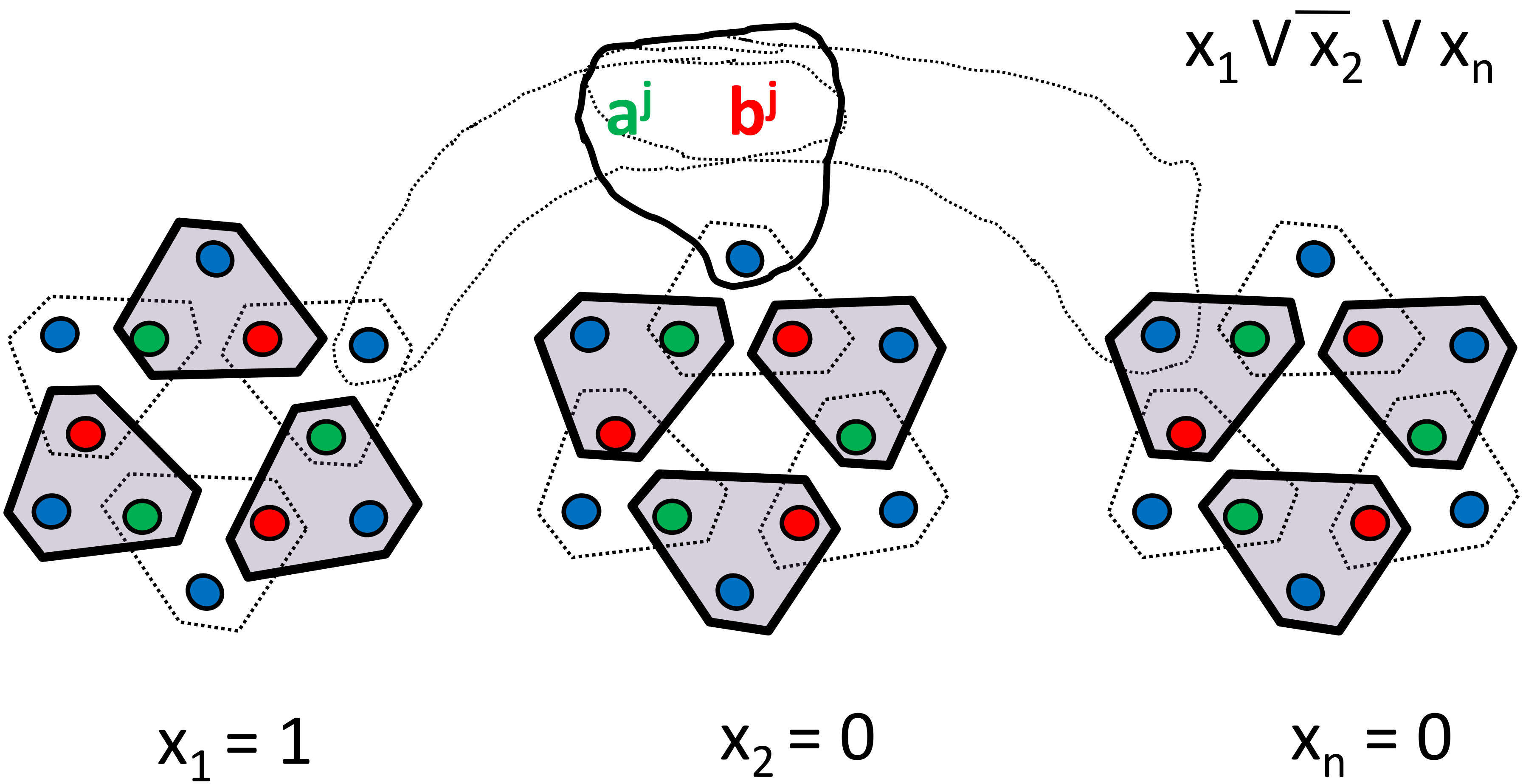

Cover all internals (A,B) once

-> truth assignment (var gadgets)

Cover all clause internal elements -> satisfies clause