Graphs and Networks **out:** November 29, 2006

Problem Set 5-Ugrad

Lecturer: Daniel A. Spielman **due:** December 14, 2006

1 Homework Policy

You may discuss the problems with other students. But, you must write your solutions independently, drawing on your own understanding. You should cite any sources that you use on the problem sets other than the textbook, TA and instructor. This means that you should list your collaborators.

You **may not** search the web for solutions to similar problems given out in other classes. If you think this policy needs any clarification, please let me know.

2 Corrections Made

1. None yet, but history suggest we will need this section.

3 Problems

1. Prove that the complete graph on 5 vertices is not planar.

2. Let $G = (V, E)$ be a 2-connected planar graph. Let $H = (F, E)$ be its dual (recall that the edges of H are in 1-1 correspondance with the edges of G). Let T be a spanning tree of G, and let S be the set of edges not in T. Prove that S is a spanning tree of H.

3. Let $\{v_1, \ldots, v_n\}$ be points in $\mathbb{R}^2$ such that no four lie on one circle. Let S be a sphere tangent to the plane $\mathbb{R}^2$ at its "south pole", and let p be its "north pole". Let Π be the stereographic projection map, which maps each point x in $\mathbb{R}^2$ to the intersection with S of the line through x and p. I claimed that this map sends circles in $\mathbb{R}^2$ to circles on S, and *vice versa*. You may assume this.

   In class, I claimed that the line segement from $\Pi(v_i)$ to $\Pi(v_j)$ is on the convex hull of the point set $\{p, \Pi(v_1), \ldots, \Pi(v_n)\}$ if and only if (v_i, v_j) is a Delaunay edge of $\{v_1, \ldots, v_n\}$.

   Prove it.

4. Let P be an infinite set of points in $\mathbb{R}^2$ such that

   a. for all $p, q \in P$, the distance between p and q is at least 1, and
   b. for every $x \in \mathbb{R}^2$, there is a $p \in P$ such that the distance from p to x is at most $\sqrt{2}$.

Let T be the Delaunay triangulation of P. Prove that there is an $\alpha > 0$ such that the smallest angle of every triangle in T is at least α.