1 Corrections Made

1. None yet, but history suggest we will need this section.

2 Problems

1. Prove that the complete graph on 5 vertices is not planar.

2. Let $G = (V, E)$ be a 2-connected planar graph. Let $H = (F, E)$ be its dual (recall that the edges of $H$ are in 1-1 correspondence with the edges of $G$). Let $T$ be a spanning tree of $G$, and let $S$ be the set of edges not in $T$. Prove that $S$ is a spanning tree of $H$.

3. Let $\{v_1, \ldots, v_n\}$ be points in $\mathbb{R}^2$ such that no four lie on one circle. Let $S$ be a sphere tangent to the plane $\mathbb{R}^2$ at its “south pole”, and let $p$ be its “north pole”. Let $\Pi$ be the stereographic projection map, which maps each point $x$ in $\mathbb{R}^2$ to the intersection with $S$ of the line through $x$ and $p$. I claimed that this map sends circles in $\mathbb{R}^2$ to circles on $S$, and vice versa. You may assume this.

In class, I claimed that the line segment from $\Pi(v_i)$ to $\Pi(v_j)$ is on the convex hull of the point set $\{p, \Pi(v_1), \ldots, \Pi(v_n)\}$ if and only if $(v_i, v_j)$ is a Delaunay edge of $\{v_1, \ldots, v_n\}$.

Prove it.

4. Let $P$ be an infinite set of points in $\mathbb{R}^2$ such that

   a. for all $p, q \in P$, the distance between $p$ and $q$ is at least 1, and
   b. for every $x \in \mathbb{R}^2$, there is a $p \in P$ such that the distance from $p$ to $x$ is at most $\sqrt{2}$.

Let $T$ be the Delaunay triangulation of $P$. Prove that there is an $\alpha > 0$ such that the smallest angle of every triangle in $T$ is at least $\alpha$.

5. Prove that every planar graph can be drawn in the plane using non-intersecting straight line segments for every edge.

   Hint: Try induction on the number of vertices. When you remove a vertex, carefully choose some edges to add between its neighbors.

   Hint: Note that it suffices to consider planar graphs in which every face is a triangle.