April 9, 2025 Iterative Solvers for Laplacian Linear Equations

Elimination: even for graph with = cn edges, can need time 2(n3) and space 2(n2) e:g. expanders

Iterative: Main operation is multiply by matrix. Only store a few vectors. Approach solution rather than compute exactly.

Goal: compute x s.t. $\|x - L^{+}b\| \in \mathbb{E}[|L^{+}b|]$ after $\int \frac{\lambda n}{\lambda 2} \ln(ls)$ multiplies (mat-vec)

Will think about solving Ax = b, A pos def. Ot for Laplacran of connected graph when $b^T 1 = 0$.

Richardson Iteration. Let A = b. C = dA = ab C = x + (aA - I)x = ab C = x = (I - dA)x + ab

Inspires iteration x==0 Xt=(I-xA)xt-1+xb

 $\frac{\text{Thm Converses if } \| \mathbb{I} - \alpha A \| < 1.}{\text{proof. Consider } x - x_t} = (\mathbb{I} - \alpha A) \times + \alpha b - (\mathbb{I} - \alpha A) \times_{t-1} - \alpha b} = (\mathbb{I} - \alpha A) (\times - \times_{t-1}) = (\mathbb{I} - \alpha A)^t (\times - \times_0) = (\mathbb{I} - \alpha A)^t \times_{t-1}$

In frect, $\|X - Xt\|_2 \leq \|I - XA\|_2^T \|X\|_2$