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Spectral Graph Theory Lecture 3.5

Dan's Favorite Inequality

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In find the following inequality very useful. We will use it often during the semester, and it might help you on the problem set.

Theorem 3.0.1. *Let $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ be positive numbers. Then*

$$\min_i \frac{a_i}{b_i} \leq \frac{\sum_i a_i}{\sum_i b_i} \leq \max_i \frac{a_i}{b_i}.$$

Proof. We have

$$\sum_i a_i = \sum_i b_i \left(\frac{a_i}{b_i}\right) \leq \sum_i b_i \left(\max_j \frac{a_j}{b_j}\right) = \left(\max_j \frac{a_j}{b_j}\right) \sum_i b_i.$$

So,

$$\frac{\sum a_i}{\sum b_i} \leq \max_j \frac{a_j}{b_j}. \tag{3.1}$$

One can similarly prove

$$\frac{\sum a_i}{\sum b_i} \geq \min_j \frac{a_j}{b_j}. \tag{3.2}$$

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