Adaptive Multidimensional Coded Modulation Over Flat Fading Channels

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***Abstract*—We introduce a general adaptive coding scheme for Nakagami multipath fading channels. An instance of the coding scheme utilizes a set of $2L$-dimensional ($2L$-D) trellis codes originally designed for additive white Gaussian noise (AWGN) channels. Any set of $2L$-D trellis codes for AWGN channels can be used. Sets for which all codes can be generated by the same encoder and decoded by the same decoder are of particular interest. A feedback channel between the transmitter and receiver makes it possible to transmit at high spectral efficiencies under favorable channel conditions and respond to channel degradation through a smooth reduction of the spectral efficiency. We develop a general technique to determine the average spectral efficiency of the coding scheme for any set of $2L$-D trellis codes. As an illustrative example, we calculate the average spectral efficiency of an adaptive codec utilizing eight 4-D trellis codes. The example codec is based on the International Telecommunications Union's ITU-T V.34 modem standard.**

***Index Terms*—Adaptive trellis coding, link spectral efficiency, Nakagami multipath fading.**

I. Introduction

The INFORMATION rate of a mobile radio link may be defined as the number of transmitted information bits per second for a given average transmit power and target bit-error-rate (BER). Future wireless networks must support file transfer and multimedia services including audio phone, video conference, video on demand, and high-speed Internet access. All these services require high information rates with short delay and low BER. Meeting these demands is challenging since radio links exhibit multipath fading which leads to time-varying degradation in the link carrier-to-noise ratio (CNR) and thus high BER.

A narrowband mobile radio link may be modeled as a single-user flat-fading channel. For a fading channel where the information rate, denoted by R [bits/s], is allowed to vary with the received CNR, the average information rate, $\langle R \rangle$ [bits/s], is obtained by averaging R over all different CNR's. If we denote the bandwidth of the channel by W [Hz], then the average information rate per unit bandwidth, called the *average spectral efficiency*, is given by $\langle R \rangle / W$ [bits/s/Hz].

Let C denote the *maximum* obtainable information rate on the fading channel for a given received CNR and arbitrarily small BER. The maximum average information rate, or Shannon capacity, for a given average transmit power and arbitrarily small BER is obtained by averaging C over all received CNR's. If we denote the Shannon capacity by $\langle C \rangle$ [bits/s], then the *maximum average spectral efficiency* (MASE) of the fading channel is $\langle C \rangle / W$ [bits/s/Hz]. The radio spectrum available for multimedia wireless communications is scarce, and a variable-rate transmission technique for future wireless networks must therefore have average spectral efficiency $\langle R \rangle / W$ close to the MASE $\langle C \rangle / W$.

Goldsmith and Varaiya [1] have determined the MASE of a general, single-user, flat-fading channel with perfect channel state information (CSI) at the transmitter and receiver for three different adaptive transmission policies: optimal simultaneous transmit power and information rate adaptation, constant power with optimal rate adaptation, and channel inversion with fixed rate. Alouini and Goldsmith [2] have since used the general theory in [1] to obtain closed-form expressions for the MASE of the three adaptive transmission policies over Nakagami multipath fading (NMF) channels. Numerical results showed that the two information rate adaptation policies yield nearly the same MASE while the channel inversion policy suffers a reduction in MASE compared to the two adaptive transmission policies.

The MASE expressions in [2] were obtained assuming continuous-rate adaptation. Goldsmith and Chua [3] have determined the average spectral efficiency of a more practical discrete-rate multilevel quadrature amplitude modulation (M-QAM) scheme in Rayleigh fading, which is a special case of NMF. They observed a gap between the obtainable average spectral efficiency and the MASE. To reduce this gap, Goldsmith and Chua [4] studied a trellis coded modulation (TCM) scheme with variable rate and variable power in Rayleigh fading. The adaptive TCM scheme utilizes a set of two-dimensional (2-D) trellis codes with different size signal constellations, and hence different spectral efficiencies, to obtain an additional coding gain relative to the uncoded adaptive M-QAM scheme. At a low target BER, the variable-rate variable-power TCM scheme exhibits a savings in average CNR of more than 20 dB compared to traditional trellis codes with fixed information rate and fixed transmit power.

In Section II, we introduce a general variable-rate constant-power TCM scheme for frequency-flat, slowly varying NMF channels. An instance of the coding scheme utilizes a set of $2L$-dimensional ($2L$-D) trellis codes where L is some positive integer. The individual codes should be designed for additive white Gaussian noise (AWGN) channels. Any set of $2L$-D trellis codes for AWGN channels can be used, but in practice only codes with different spectral efficiencies are of interest. For

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$L = 1$, the coding scheme reduces to the 2-D coding scheme in [4] with constant transmit power. We concentrate on the multidimensional $(L \geq 2)$ versions of the coding scheme.

The hardware complexity of the coding scheme is significantly reduced if all codes can be generated by the same encoder and decoded by the same Viterbi decoder. Unlike other fading resistant coding schemes based on trellis codes for AWGN [5], [6], the scheme considered here does not utilize interleaving. Good performance of the scheme requires accurate estimates of the CSI at the decoder and a reliable feedback channel between the encoder and the decoder. Buffering of the input data is also needed since the information rate varies with the channel conditions.

In Section III we develop a general technique to calculate the average spectral efficiency $\langle R \rangle / W$ of the coding scheme for any set of $2L$-D trellis codes. As an illustrative example, we determine the average spectral efficiency of a certain variable-rate encoder and decoder (codec) in Section IV. The example codec is based on the International Telecommunications Union's ITU-T V.34 modem standard. The codec utilizes eight nested M-QAM signal constellations containing 4, 8, 16, 32, 64, 128, 256, and 512 signal points to encode and decode eight 4-D trellis codes. The average spectral efficiency $\langle R \rangle / W$ is compared to the MASE $\langle C \rangle / W$. A summary and a discussion are presented in Section V.

II. System Model and Coding Scheme

The end-to-end system for variable-rate constant-power TCM over NMF channels was first studied in [1]–[4] and [7]. It is assumed that the time-varying phase shift and channel gain (fading envelope) of the transmitted signal are perfectly known at the receiver. Consequently, the receiver is able to fully compensate for the phase variation, i.e., we assume perfect coherent detection. A decision device utilizes the channel gain to select an appropriate information rate by choosing a code among a set of $2L$-D trellis codes. The decision device then informs the transmitter about its decision via a feedback channel.[1]

As mentioned in the Introduction, the $2L$-D trellis codes should in practice be generated by the same encoder and decoded by the same Viterbi decoder. Hence, we assume that the trellis codes are implemented as a variable-rate TCM codec where it is possible to change rate at any time.

For satisfactory operation of the communication system, both the variable-rate encoder and the variable-rate Viterbi decoder must use the same code at any instant. An efficient error control scheme is therefore needed to ensure an error-free feedback channel. We simply assume that the feedback channel is noiseless.

Let the transmitted 2-D signal have complex baseband representation $x(t)$ at time $t \in \{1, 2, \cdots\}$. The stationary and ergodic channel gain, represented by $\alpha(t)\,(\geq 0)$, is independent of the channel input and has the same value in both signal dimensions. The received baseband signal is given by $y(t) = \alpha(t)x(t) + n(t)$ where $n(t)$ denotes complex AWGN. The real and imaginary parts of the noise are statistically independent, both with variance $(WN_0)/2$ where $N_0/2$ [W/Hz] is the two-sided noise power spectral density.

Let S [W] denote the constant transmit signal power. The instantaneous received CNR of the system is equal to $\gamma(t) = S\alpha^2(t)/(N_0 W)$, and its expected value is $\overline{\gamma} = S\Omega/(N_0 W)$ where $\mathrm{E}[\alpha^2(t)] = \Omega$ is the average received power gain [1], [3]. In the remainder of the paper we will omit the time reference t and refer to α and γ.

The real-valued channel gain α is modeled as a continuous stochastic variable with a Nakagami probability density function (pdf) controlled by the Nakagami fading parameter, a real number $m \geq 1/2$. The parameter m is here assumed to be a positive integer. The Nakagami distribution represents a wide range of multipath fading channels via different integer values of m. When $m = 1$ the pdf is the Rayleigh pdf. For $m = 2, 3, \ldots,$ the Nakagami distribution closely approximates the Rice distribution [8, p. 48].

The instantaneous received CNR γ on an NMF channel is a continuous stochastic variable with gamma distribution. The PDF for γ is given by [2], [7]

$$p_\gamma(\gamma) = \left(\frac{m}{\overline{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-m\,\frac{\gamma}{\overline{\gamma}}\right), \qquad \gamma \geq 0 \tag{1}$$

where $\Gamma(m)$ is the *gamma function* which equals $\Gamma(m) = (m-1)!$ when m a positive integer. The fading becomes less severe when m increases because the variance of γ, $\sigma_\gamma^2 = (\overline{\gamma})^2/m$, is reduced.

The delay in the feedback channel of the system model may reduce the achievable average spectral efficiency for a given target BER when the channel gain α or, equivalently, the received CNR γ changes rapidly [9]. We assume that α changes at a rate much slower than the symbol rate, so that the channel remains roughly constant over several hundred symbols, and we therefore ignore the propagation delay.

The MASE of the system model with constant transmit power and a delay-free feedback channel is

$$\frac{\langle C \rangle}{W} = \log_2(e) e^{m/\overline{\gamma}} \sum_{k=0}^{m-1} \left(\frac{m}{\overline{\gamma}}\right)^k \Gamma\left(-k,\, \frac{m}{\overline{\gamma}}\right) \qquad \text{[bits/s/Hz]} \tag{2}$$

for m a positive integer [2, eq. (23)]. Here $\Gamma(\cdot, \cdot)$ is the *complementary incomplete gamma function* defined by [10, eq. (8.350.2), p. 949]

$$\Gamma(\beta,\, x) = \int_x^\infty t^{\beta-1} e^{-t}\, dt.$$

The function is commonly available in numerical software.

Assume that N quantization levels (or *fading regions*) are used to represent the instantaneous received CNR γ. The theoretical transmission scheme that achieves MASE has $N = \infty$ infinitely small fading regions [1]. Our coding strategy for a finite number of fading regions is to assign one $2L$-D trellis code to each fading region. The trellis codes are introduced to compensate for the finite quantization of γ and the limitations on the coding alphabet associated with realizable signal constellations.

Let the $N\,(<\infty)$ regions be defined by the thresholds $\gamma_1 < \gamma_2 < \cdots < \gamma_{N+1}$. Code n, $n \in \{1, 2, \cdots, N\}$, is used when

[1]We remark that the protocols needed to change between the different codes are not considered explicitly, but are assumed to work perfectly.

the instantaneous received CNR γ falls in region n, i.e., when $\gamma_n \leq \gamma < \gamma_{n+1}$. We assume that fading region 1 represents the smallest values of γ for which information is transmitted. When $\gamma < \gamma_1$, no information is sent.

We use trellis codes for AWGN channels. To understand why, observe that the width $\gamma_{n+1} - \gamma_n$ of each fading region goes to zero when the number of fading regions N goes to infinity. Consequently, for large N the received CNR may be approximated by a constant for each fading region and we may view the NMF channel at a given time instant as an element in a set of AWGN channels with different but constant CNR's.

The $2L$-D signal constellation for code n is given by the L-fold Cartesian product of a 2-D signal set with $M_n = 2^{k_n}$ signal points (or symbols). Each time the encoder for code n receives $p = L \cdot k_n - 1$ information bits, it generates $p+1 = L \cdot k_n$ coded bits. The coded bits then determine L transmittable 2-D modulation symbols.

If the time T between the transmission of two consecutive 2-D symbols is such that there is no intersymbol interference in the samples at the output of the channel, then the information rate for code n is equal to $R_n = (\log_2(M_n) - 1/L)/T = (k_n - 1/L)/T$ [bits/s]. Since the Nyquist bandwidth is $W = 1/T$, the maximum spectral efficiency for code n is $R_n/W = k_n - 1/L$ [bits/s/Hz]. The codes should have spectral efficiencies $k_n - 1/L$ which increase with γ, i.e., $k_n < k_{n+1}$ for $n = 1, 2, \cdots, N-1$. This makes it possible to transmit at high information rates when there is little or no fading, and to reduce the information rate as the fading increases.

III. Special Efficiency of the Coding Scheme

In this section we determine a general expression for the average spectral efficiency of the coding scheme. The expression is valid for any set of $2L$-D trellis codes.[2]

It is first argued that the BER of a $2L$-D trellis code on an AWGN channel can be approximated by an expression of the form

$$\text{BER} \approx a \cdot \exp\left(-\frac{b\gamma^*}{M}\right) \tag{3}$$

where γ^* is the CNR and the constants a and b depend on the weight distribution of the code.

The encoder for the code generates L 2-D modulation symbols each time it receives p information bits. The 2-D signal constellation consists of $M = 2^k$ signal points drawn from a regular lattice where the minimum Euclidean distance between two points is d_0. When k is even, we assume that the constellation is quadratic, otherwise the constellation has a "cross shape" as described in [11]. The Voronoi region of a signal point is a square with area d_0^2. The average transmit energy can be expressed as Ed_0^2 where E is some positive real number.

To obtain an expression of the desired form (3) for equiprobable signaling, we adopt the integral approximation technique of [11] to express E in terms of M. When k is even, the square constellation has length $2^{k/2} d_0$ resulting in a total area of $2^k\, d_0^2 = Md_0^2$. The average energy of the M signal points is approximately equal to the average energy $\overline{x^2 + y^2}$ of all points (x, y) inside the constellation area. This integral approximation gives average energy equal to

$$\frac{1}{Md_0^2} \int_{-2^{k/2-1}d_0}^{2^{k/2-1}d_0} \int_{-2^{k/2-1}d_0}^{2^{k/2-1}d_0} (x^2 + y^2)\, dx\, dy = (M/6)d_0^2.$$

When k is odd, the approximation is equal to $(31/32)(M/6)d_0^2$. In both cases, the average energy is approximated by an expression of the form $b' M\, d_0^2$ for b' a positive number.[3]

The minimum squared Euclidean distance of the $2L$-D code can likewise be expressed as $d_E^2 = \beta d_0^2$ for some $\beta > 1$. While the instantaneous CNR is a random variable in the case of a fading channel, the CNR for an AWGN channel is a constant $\gamma^* = Ed_0^2/N_0$. For a high CNR, the BER can be approximated by

$$\text{BER} \approx \frac{w_{d_E}}{p}\, Q\left(\sqrt{\frac{d_E^2}{2}\gamma^*}\right) \tag{4}$$

where w_{d_E} is the average total number of information bit errors associated with the codewords in distance d_E from the correct codeword [12, eq. (14–21), p. 376].

We now set the average transmit energy $Ed_0^2 = b' M d_0^2 = c$ where c is an arbitrary positive constant. It follows that $d_0^2 = c/(b'M)$. If we then use the upper bound $Q(y) \leq 1/2 \exp(-y^2/2)$ and rewrite $d_E^2 = \beta d_0^2 = (\beta c)/(b'M)$, we have from (4) that

$$\text{BER} \approx \frac{w_{d_E}}{2p} \exp\left(-\frac{\beta c}{4b'}\gamma^*/M\right)$$

which has the desired form (3) for $a = w_{d_E}/(2p)$ and $b = (\beta c)/(4b')$.

We have only shown that the approximation in (3) is good for a high CNR. However, plots of BER found in the literature strongly indicate that we can use curve fitting techniques to determine values for a and b such that the expression can be used with good accuracy also for medium and low CNR's. We will illustrate this in Section IV.

We are now ready to determine a general expression for the average spectral efficiency of the adaptive coding scheme on an NMF channel. The coding scheme utilizes N trellis codes whose 2-D signal constellations have $M_n = 2^{k_n}$ signal points and whose BER can be approximated by (3) for some values $a = a_n$ and $b = b_n$ for $n = 1, \ldots, N$. Code n is to be used when the instantaneous received CNR γ falls in fading region n, i.e., $\gamma_n \leq \gamma < \gamma_{n+1}$. The lower threshold γ_n is equal to the least CNR required to achieve a given target BER denoted by BER_0. We obtain the following thresholds by assuming equality in (3), substituting γ_n, a_n, b_n, M_n, BER_0, and inverting the expression with respect to the CNR:

$$\begin{aligned} \gamma_n &= (M_n K_n)/b_n, \qquad n = 1, 2, \ldots, N \\ \gamma_{N+1} &= \infty \end{aligned} \tag{5}$$

where $K_n = -\ln(\text{BER}_0/a_n)$.

It may not be completely obvious that $\gamma_n < \gamma_{n+1}$ for $n = 1, 2, \ldots, N$ since a_n and b_n vary with n, but this monotonicity

[2]We remark that the following analysis does not require that all trellis codes are generated by the same encoder and decoded by the same decoder.

[3]The integral approximations are calculated in [11] for $d_0 = 2$.

follows from the fact that, for a given BER, the minimum required CNR for code $n+1$ is larger than the required CNR for code n when $M_{n+1} > M_n$. In practice, $0 < \mathrm{BER}_0 < a_n$ such that $K_n > 0$ and all thresholds $\gamma_n > 0$.

The average spectral efficiency $\langle R \rangle / W$ is equal to the sum over all n of the spectral efficiencies $R_n/W = k_n - 1/L$ of the individual codes, each weighted by the probability P_n that the CNR γ falls in region n

$$\frac{\langle R \rangle}{W} = \sum_{n=1}^{N} \frac{R_n}{W} \cdot P_n = \sum_{n=1}^{N} (k_n - 1/L) P_n \qquad \text{[bits/s/Hz]}. \tag{6}$$

The probabilities $P_n = \int_{\gamma_n}^{\gamma_{n+1}} p_\gamma(\gamma)\, d\gamma$ where $p_\gamma(\gamma)$ is defined by (1), are given by [7, eq. (10)]

$$P_n = \frac{\Gamma\left(m, \frac{m\gamma_n}{\overline{\gamma}}\right) - \Gamma\left(m, \frac{m\gamma_{n+1}}{\overline{\gamma}}\right)}{(m-1)!}. \tag{7}$$

For the special case of m a positive integer, no numerical software program is needed to calculate $\Gamma(\cdot, \cdot)$ in (7) since there exists a closed-form expression [13, eq. (11.6), p. 278]

$$\Gamma(m, \mu) = (m-1)! e^{-\mu} \sum_{i=0}^{m-1} \frac{\mu^i}{i!} \qquad m = 1, 2, 3, \ldots, \qquad \mu > 0. \tag{8}$$

It follows from (8) that (7) is given by $P_n = \exp(\gamma_n/\overline{\gamma}) - \exp(-\gamma_{n+1}/\overline{\gamma})$ for $m = 1$ (Rayleigh fading).

IV. Spectral Efficiency of Example Codec

In the following, we calculate the average spectral efficiency of a particular instance of the adaptive coding scheme.

Using techniques described in [4] and [14], we designed an adaptive codec with eight 4-D trellis codes utilizing the eight nested 2-D signal constellations in Fig. 1. At each *even* time instance $2t$, the variable-rate encoder receives $p(2t)$ information bits, $p(2t) \in \{3, 5, 7, 9, \ldots, 17\}$, and generates $p(2t)+1$ coded bits. The overall encoder contains the rate 2/3 trellis encoder and the bit converter, but not the 4-D block encoder, depicted in [14, Fig. 6]. The rate 2/3 trellis encoder and bit converter are also used in the International Telecommunications Union's ITU-T V.34 modem standard [15].

The $p(2t)+1$ coded bits define two symbol numbers, each in the range $0, 1, \ldots, 2^k - 1$, $k = (p(2t)+1)/2$, which is the range of the symbol numbers for the 2-D constellation with $M = 2^k$ symbols. The two chosen symbols are sent through the channel at times $2t$ and $2t+1$.

The number of information bits $p(2t)$ shifted into the encoder determine which 2-D constellation and hence which code is used. The instantaneous spectral efficiency $p(2t)/2$ [bits/s/Hz] may therefore be varied simply by changing the number of information bits shifted into the encoder. Since the overall encoder generates two symbols at the time, the spectral efficiency may only be changed at even time instants.

All codes have $d_E^2 = 4d_0^2$ and trellises with 16 states and $2^{p(2t)-2}$ parallel branches. The Viterbi decoder for the code with $p = 17$ is used to decode the codes with $p < 17$ since these codes are all contained in the $p = 17$ code. A rough estimate of the scheme's hardware complexity is therefore given by the hardware complexity of the largest $(p = 17)$ code.

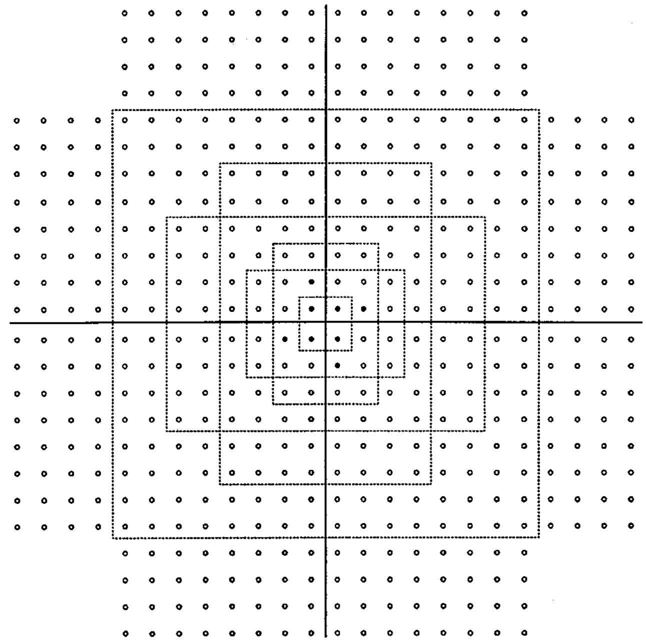

Fig. 1. Nesting of the 4-QAM, 8-STAR, 16-QAM, 32-CROSS, 64-QAM, 128-CROSS, 256-QAM, and 512-CROSS signal constellations. The filled black circles constitute the 8-STAR constellation.

The individual codes' BER performances on an AWGN channel were simulated for various CNR's. The simulation points (represented by boxes) are shown in Fig. 2. For each CNR, the simulation program generated at least 200 decoded bits in error before calculating the BER. All simulation points were calculated using a relatively short path memory of length 9. The true squared Euclidean distance was used as metric in the Viterbi decoder.

To calculate the average spectral efficiency of the example codec, let the nth 4-D trellis code, $n \in \{1, 2, \ldots, 8\}$, be the code based on the 2-D signal constellation in Fig. 1 with $M_n = 2^{n+1} \in \{4, 8, 16, 32, 64, 128, 256, 512\}$ signal points. The BER for code n can be approximated by the expression

$$\mathrm{BER}_n \approx a_n \exp\left(\frac{-b_n \gamma}{M_n}\right). \tag{9}$$

Curve fitting with the least squares method [16, Ch. 10] is used to obtain the parameters a_n and b_n listed in Table I. The resulting BER approximations (9) are plotted in Fig. 2. Note that the approximated BER curves are very close to the simulated BER points for all codes. The thresholds γ_n, calculated from (5), are also tabulated in Table I. The average spectral efficiency, obtained from (6) and (7), is plotted in Fig. 3 for Nakagami fading parameters $m = 1, 2, 4$ and average received CNR $12 \le \overline{\gamma} \le 26$ [dB].

The difference between the MASE (2) and the average spectral efficiency is shown in Fig. 4. Observe that the difference decreases as $\overline{\gamma}$ increases. One reason for this may be that the 8-STAR signal constellation in Fig. 1 has a rather large average

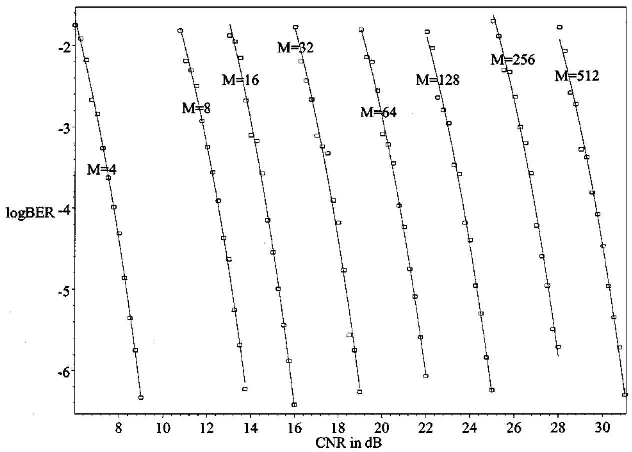

Fig. 2. The boxes are BER estimates obtained from software simulation and the curves are estimates obtained from (9).

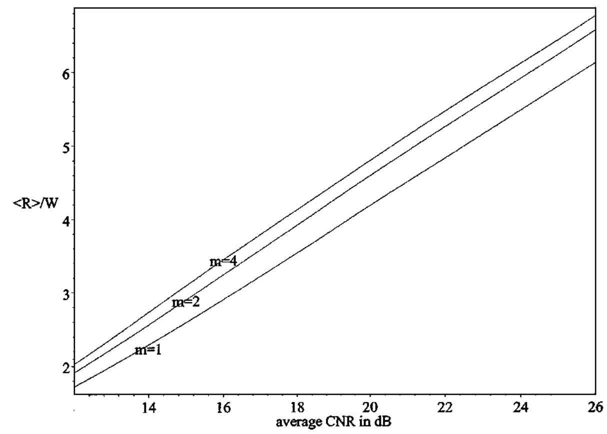

Fig. 3. Average spectral efficiency $\langle R\rangle/W$ (bits/s/Hz) of example codec for target $\text{BER}_0 = 10^{-3}$.

TABLE I
PARAMETERS a_n AND b_n, FOR EXAMPLE, CODEC AND CALCULATED THRESHOLDS γ_n (dB) FOR TARGET $\text{BER}_0 = 10^{-3}$

n	M_n	a_n	b_n	γ_n [dB]
1	4	896.0704	10.7367	7.1
2	8	404.4353	6.8043	11.8
3	16	996.5492	8.7345	14.0
4	32	443.1272	8.2282	17.0
5	64	296.6007	7.9270	20.1
6	128	327.4874	8.2036	23.0
7	256	404.2837	7.8824	26.2
8	512	310.5283	8.2425	29.0

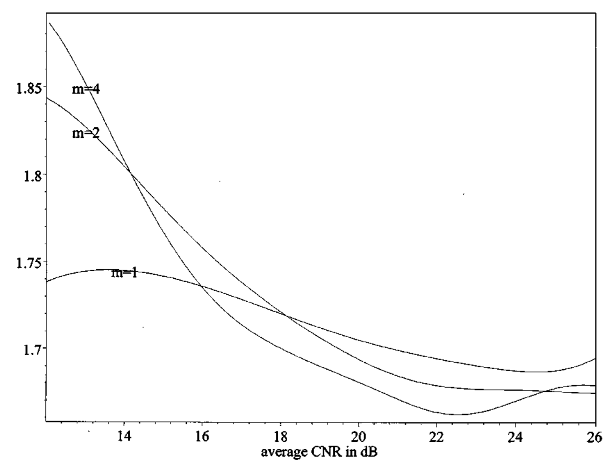

Fig. 4. Difference between MASE $\langle C\rangle/W$ (bits/s/Hz) and average spectral efficiency $\langle R\rangle/W$ (bits/s/Hz) of example codec for target $\text{BER}_0 = 10^{-3}$.

energy[4] equal to $3/2d_0^2$. Hence, the code based on the 8-STAR constellation requires a rather large received CNR γ to achieve the target BER. As $\overline{\gamma}$ increases, the probability that this sub-optimal code is used decreases, and its negative effect on the average spectral efficiency diminishes.

V. DISCUSSION AND SUMMARY

We have introduced a general adaptive coding scheme for single-user channels with frequency-flat, slowly varying NMF. An instance of the scheme utilizes N $2L$-D trellis codes originally designed for AWGN channels. A general expression for the coding scheme's average spectral efficiency was obtained.

As an example, we approximated the average spectral efficiency of a codec containing eight 4-D trellis codes (see Fig. 3). The average spectral efficiency may be increased if we use more circular versions of the large signal constellations. A moderate shaping gain may also be obtained by utilizing extended constellations and a simple block code [14]. By turning the shaping on and off, it is possible to increase the spectral efficiency in steps of 0.5 [bits/s/Hz] instead of 1 [bits/s/Hz] as has been done in this paper.

The example codec utilizes eight nested 2-D signal constellations with four or more signal points (see Fig. 1). The relatively large average energy of the 8-STAR constellation resulted in a larger difference between the MASE and the obtained average spectral efficiency for small CNR's (see Fig. 4). The extra loss in average spectral efficiency incurred by the 8-STAR code is not dramatic. However, we do not recommend that it is used in a real codec.

An interesting alternative to our example codec is a codec based on Ungerboeck's 2-D trellis codes with 16 states [17].[5] These codes have $d_E^2 = 6d_0^2$ while our codes have $d_E^2 = 4d_0^2$. Unfortunately, Ungerboeck's codes have only 180° rotational invariance. Our codes, on the other hand, can be made transparent to 90°, 180°, and 270° phase rotations of the signal constellations by introducing a differential encoder and a differential decoder. Hence, the example codec eliminates the need for a coherent phase reference [18].

Alouini and Goldsmith [2] showed that as long as the information rate can be varied, only a very modest increase in MASE is obtained by also varying the transmit power. They concluded that the information rate adaption rather than the transmit power adaption is the key to increasing the average spectral efficiency on NMF channels. However, even though very little is gained

[4]There exist other constellations with 8 points that have smaller average energy than 8-STAR. Unfortunately, these constellations are not subconstellations of the larger constellations in Fig. 1.

[5]Variable-rate codecs containing Ungerboeck codes with 4, 8, 32, and 128 states were investigated in [4].

in terms of MASE by varying the transmit power, this does not tell us anything about how to best achieve an average spectral efficiency close to the MASE. An informal comparison of plots in this paper and plots in [4] for $m = 1$ (Rayleigh fading) indicates that more codes are needed to achieve a given average spectral efficiency when the transmit power is fixed than when the power is allowed to vary.

The above result for 4-D codes, as well as the results obtained by Goldsmith and Chua [4] for 2-D codes, indicate that, for roughly the same hardware complexity, variable-rate TCM may achieve higher average spectral efficiency than many fixed-rate trellis codes. However, before these spectral efficiencies can be obtained in practice, more work is needed to determine new channel estimation techniques that will allow the adaptive coding scheme to meet BER specifications with imperfect channel estimation and nonzero feedback delay. Some interesting preliminary results in this direction may be found in [19].

We end this paper with a comment on the time delay in the feedback channel. Goeckel [20] has recently shown that the output BER of an adaptive coding scheme may increase rapidly with growing delay in the feedback channel. We have assumed that the feedback delay is zero. More work is therefore needed to determine the BER performance of our adaptive coding scheme for nonzero feedback delay.

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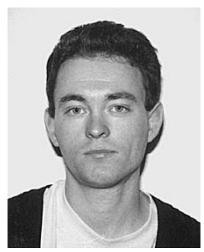

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