Algorithms, April 7, 2011

Today

Will prove more problems are NP-complete:

      3D Matching
      Generalized 3DM
      Exact Cover
      Subset Sum
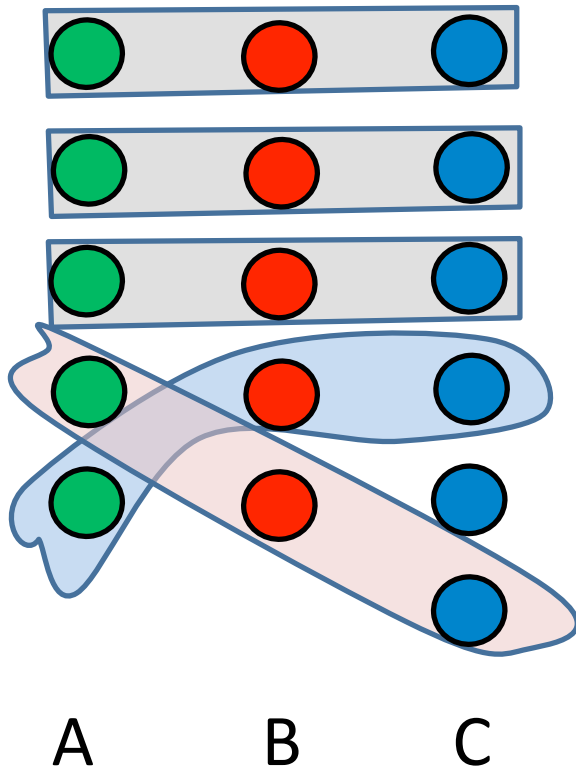   Interval Sched with Deadlines
           and Release Times (ISDR)

Generalized 3DM

Given three sets, A, B, C, $|A| = |B| = k$
   and triples $T_1, ..., T_n$, each with one element of A, B, and C

Do there exist k pairwise disjoint triplets?

Equivalent: disjoint triplets
   that cover all of A and B.

A   B   C

Gen-3DM is NP-Complete

Clearly in NP, because can check a proposed matching.
To prove NP-hard, will show 3-SAT $\leq_P$ Gen-3DM.

Given an collection of clauses $C_1, ..., C_k$, each with
at most 3 terms, on variables $x_1, ..., x_n$

produce sets A, B, C, and triples $S_1, ..., S_m$
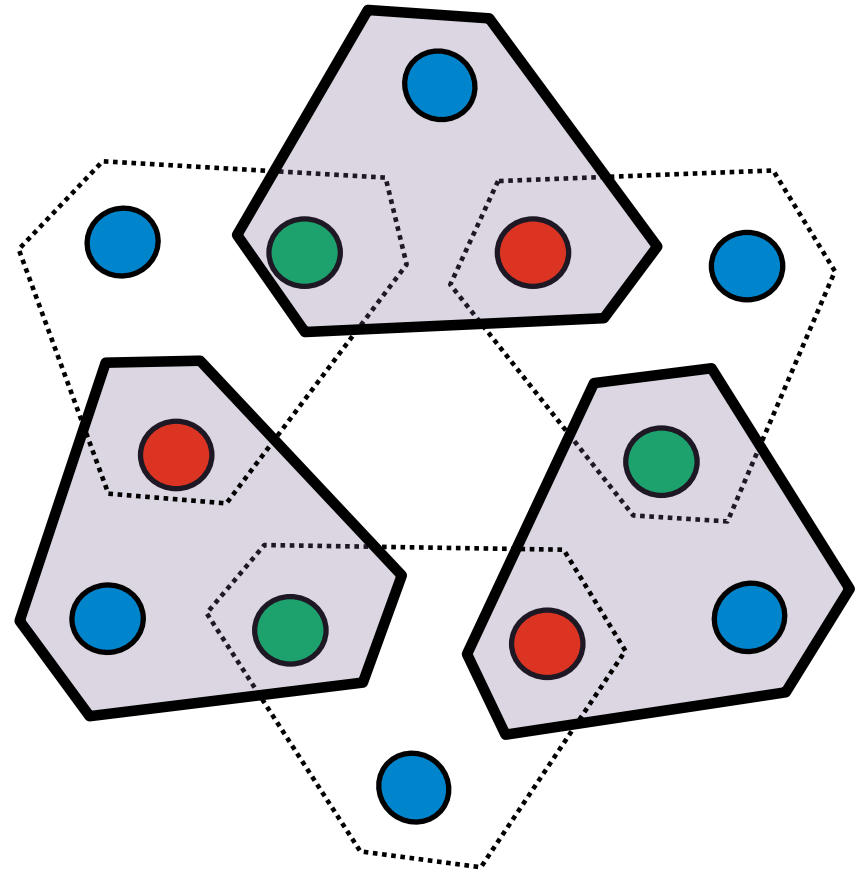that have matching iff the clauses are all satisfiable

Gen-3DM NP-Complete – variable gadgets

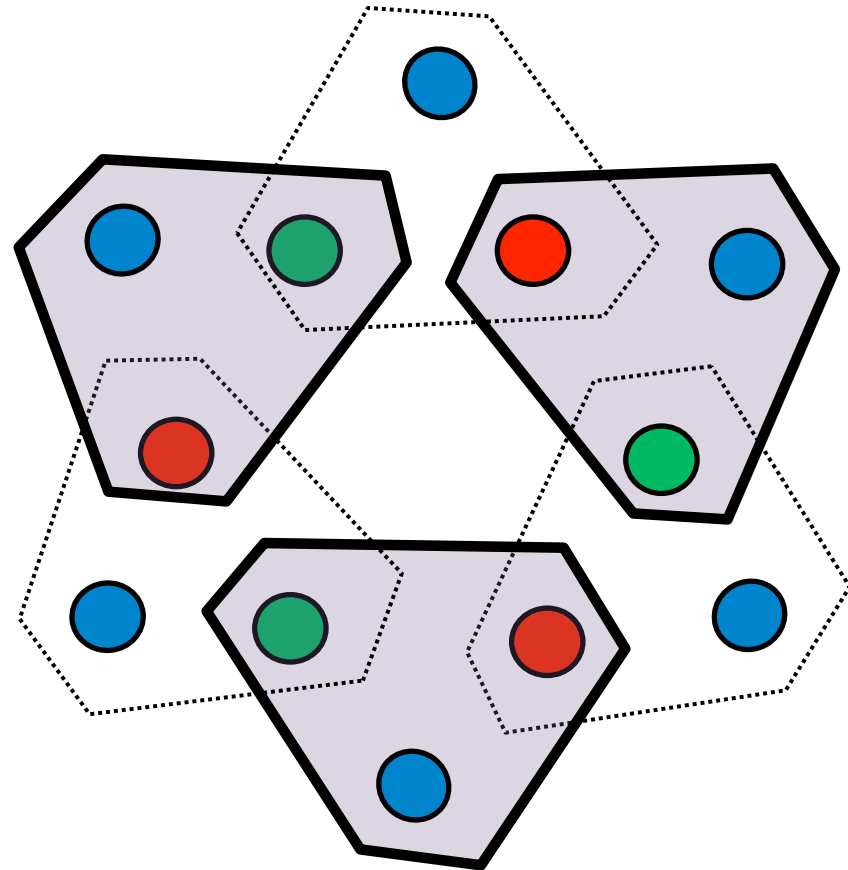If these are only triples containing inner elements, must cover by all odd or all even triples
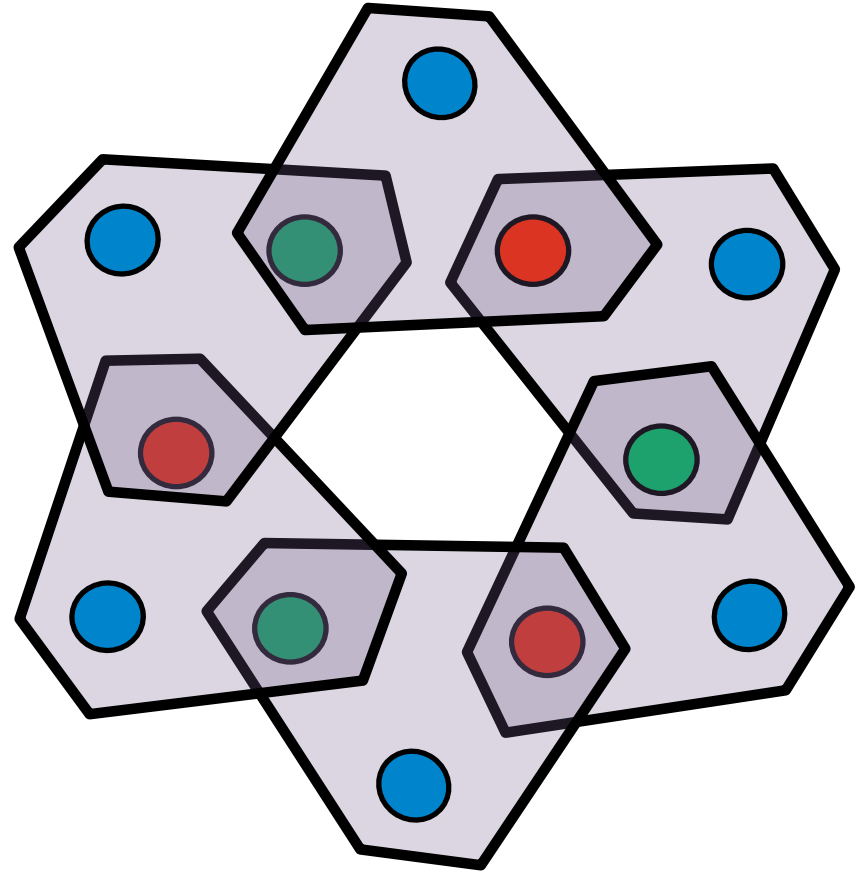
Gen-3DM NP-Complete – variable gadgets

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Gen-3DM NP-Complete – variable gadgets

If these are only triples containing inner elements, must cover by all odd or all even triples

3DM NP-Complete – variable gadgets

For variable x_i in d clauses, create gadget with 2d inner elements:

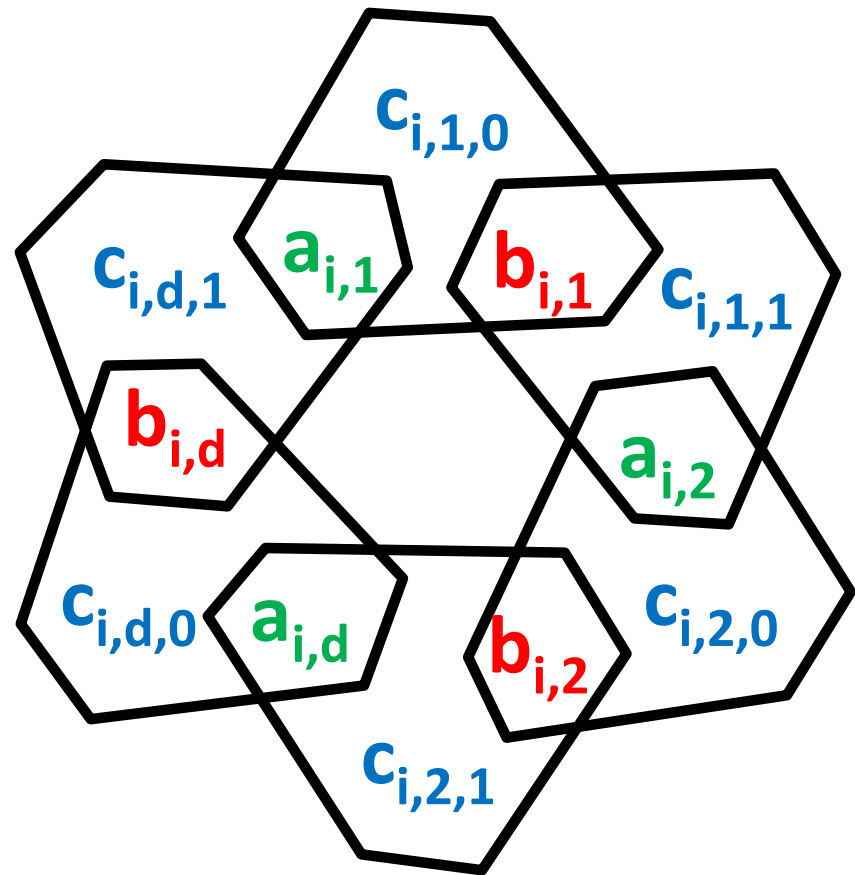$a_{i,1}, a_{i,2}, ..., a_{i,d}$
$b_{i,1}, b_{i,2}, ..., b_{i,d}$

and 2d outer elements

$c_{i,1,0}, c_{i,2,0}, ..., c_{i,d,0},$
$c_{i,1,1}, c_{i,2,1}, ..., c_{i,d,1}$
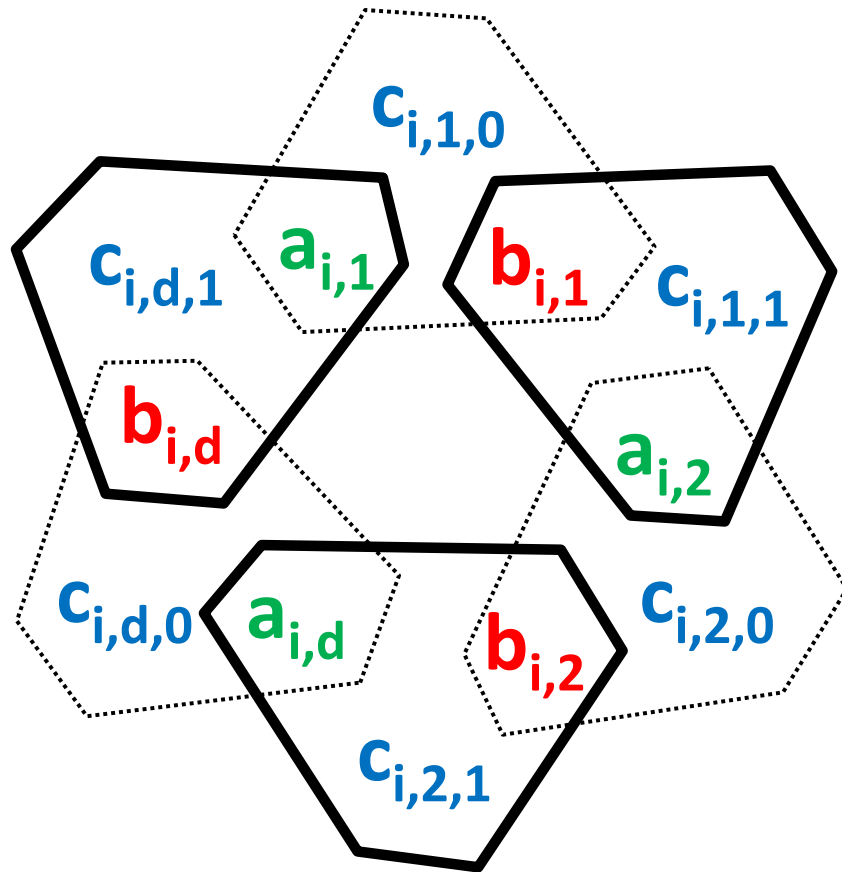
and triples as shown:

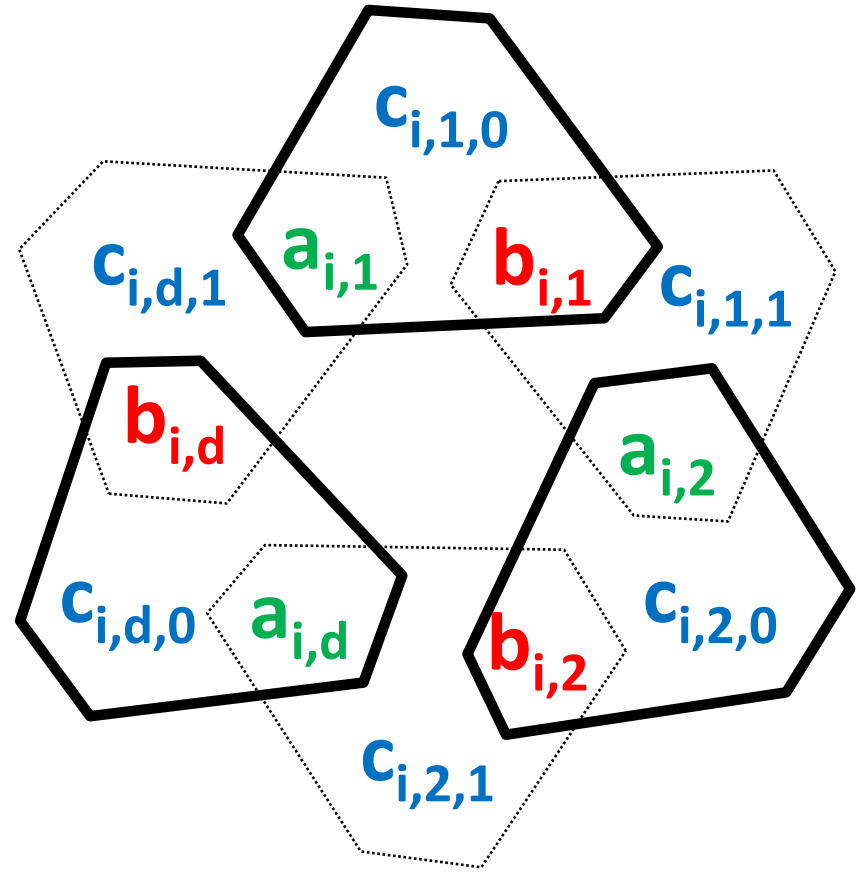$(a_{i,k}, b_{i,k}, c_{i,k,0}), (a_{i,k+1}, b_{i,k}, c_{i,k,1})$

3DM NP-Complete – variable gadgets

Interpret covering inner elements by odd sets as false.

Interpret covering inner elements by even sets as true
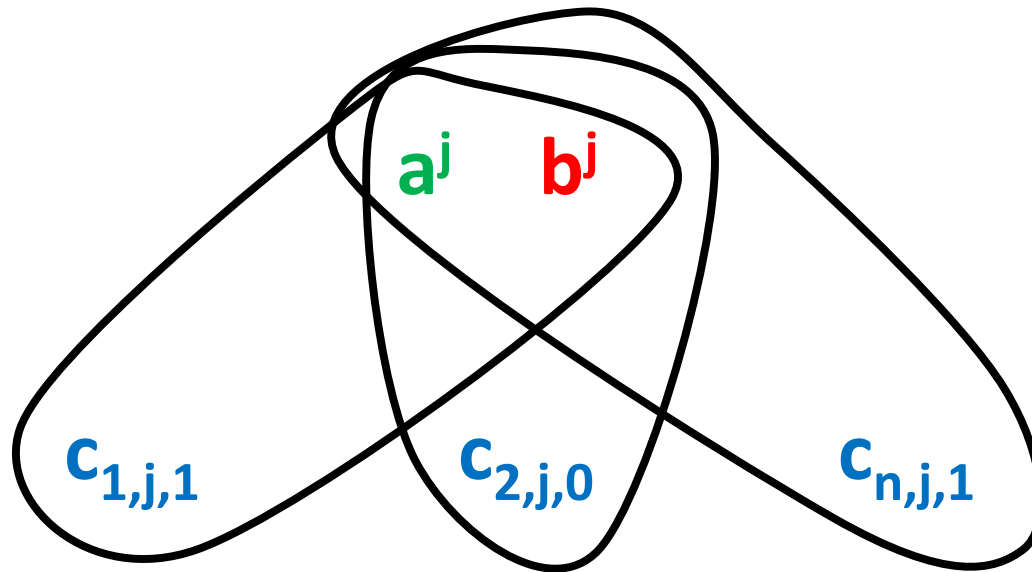

Expose $c_{i,*,0}$

Expose $c_{i,*,1}$

3DM NP-Complete – clause gadget

Say clause Cj has form $x_1 \lor \overline{x_2} \lor x_n$

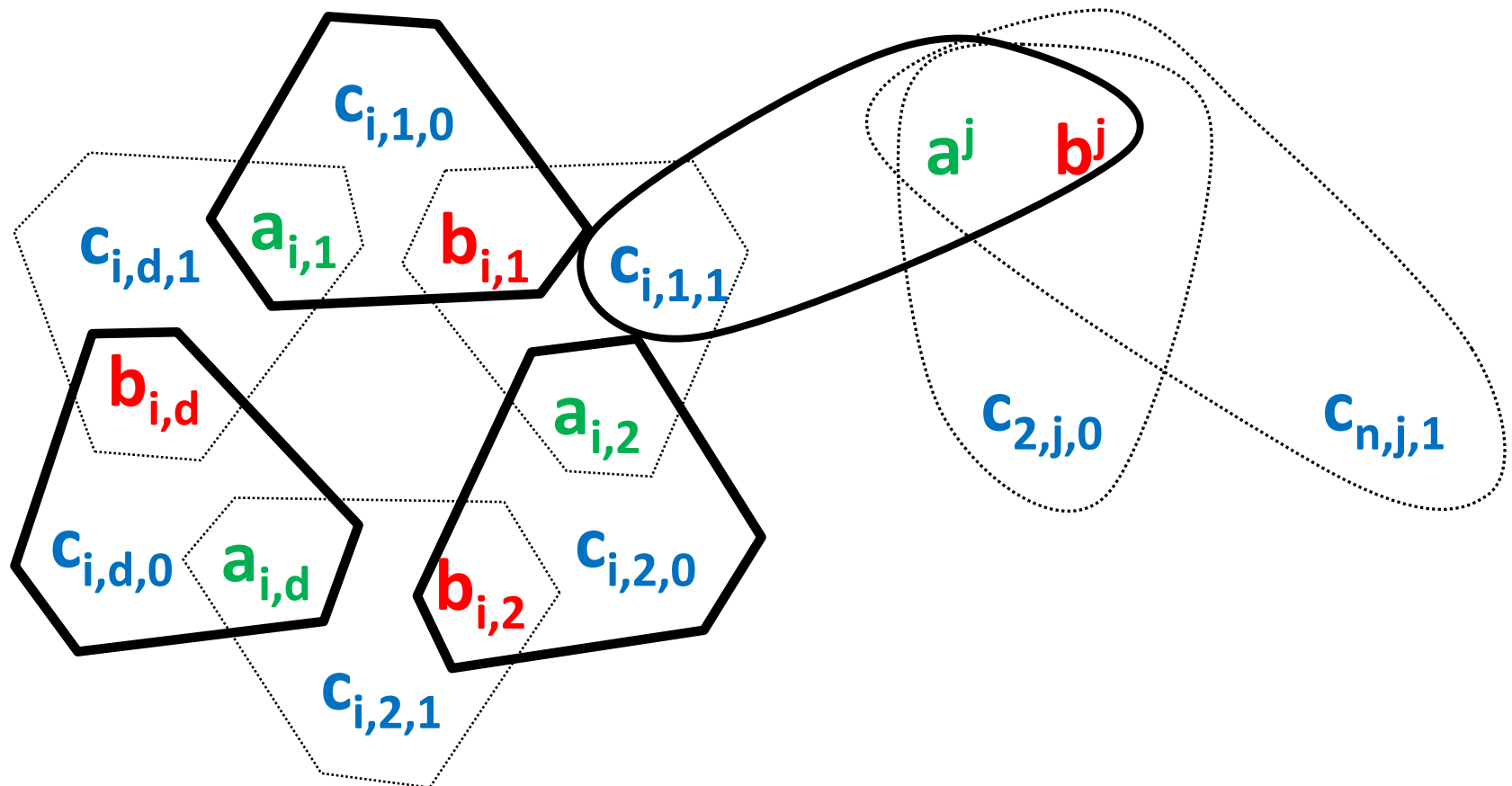Create two elements for the clause:

a^j and b^j

and create triples with these and terms that satisfy clause : $(a^j , b^j c_{1,j,1})$, $(a^j , b^j c_{2,j,0})$, $(a^j , b^j c_{n,j,1})$,

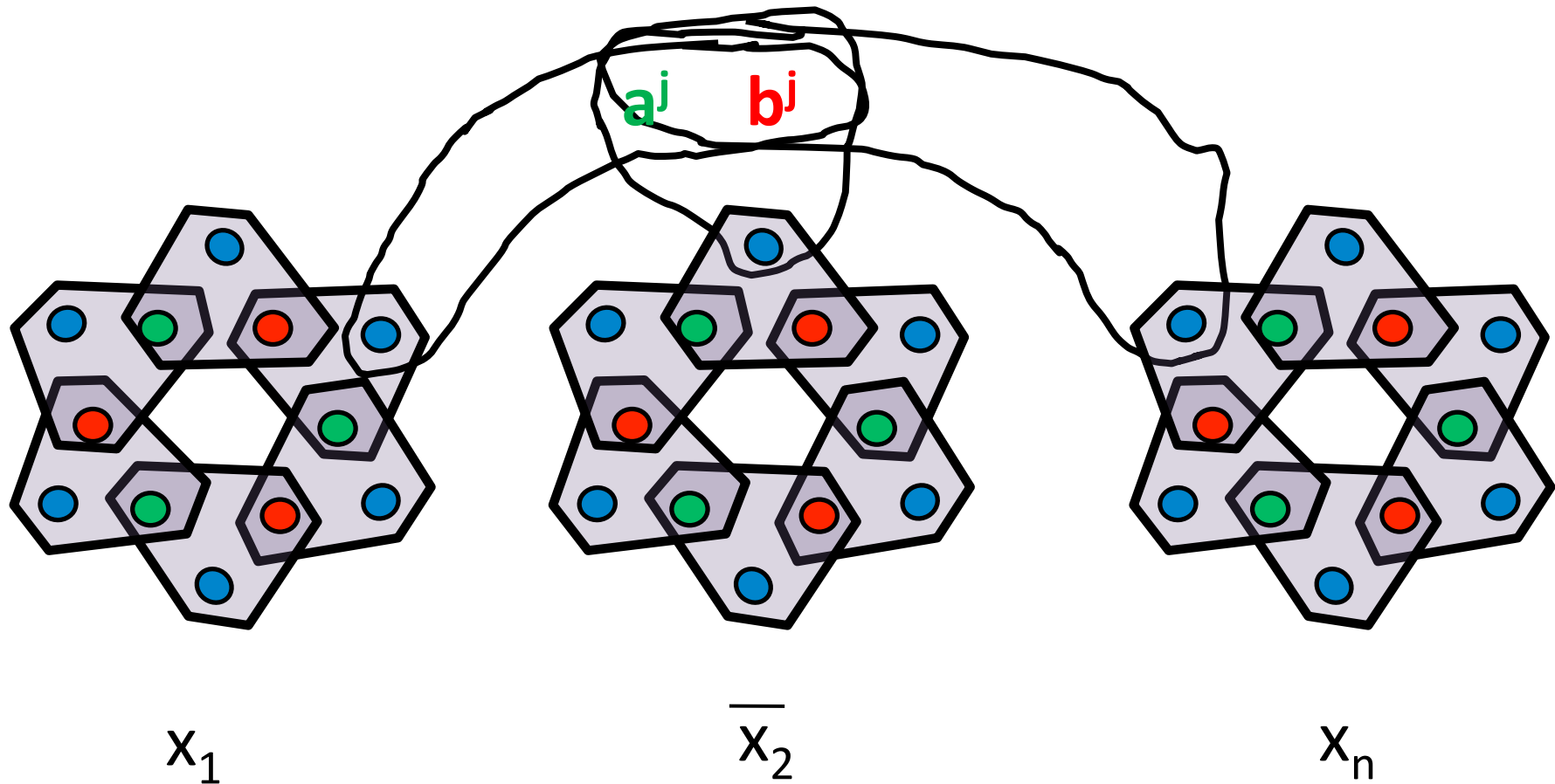# 3DM NP-Complete – clause gadget

Say clause Cj has form $x_1 \lor \overline{x_2} \lor x_n$

If these are only triples with the clause elements, must cover by a variable's external element that satisfies clause, and variable gadgets enforce consistency

3DM NP-Complete – clause gadget

Say clause Cj has form $x_1 \lor \overline{x_2} \lor x_n$

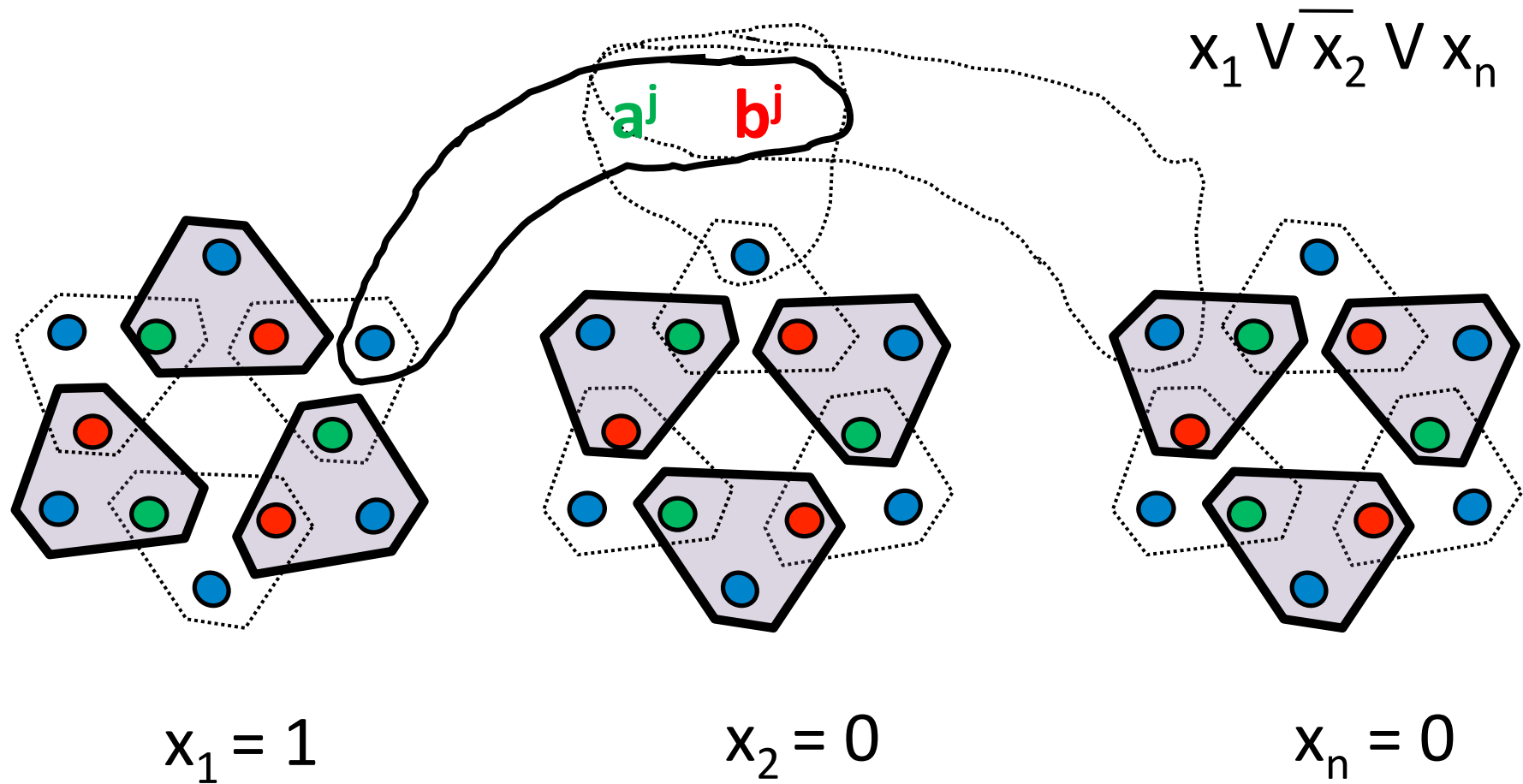x_1 $\overline{x_2}$ x_n

Each clause gets own external element for each variable

Truth assignment -> choice of triples at variable gadgets.
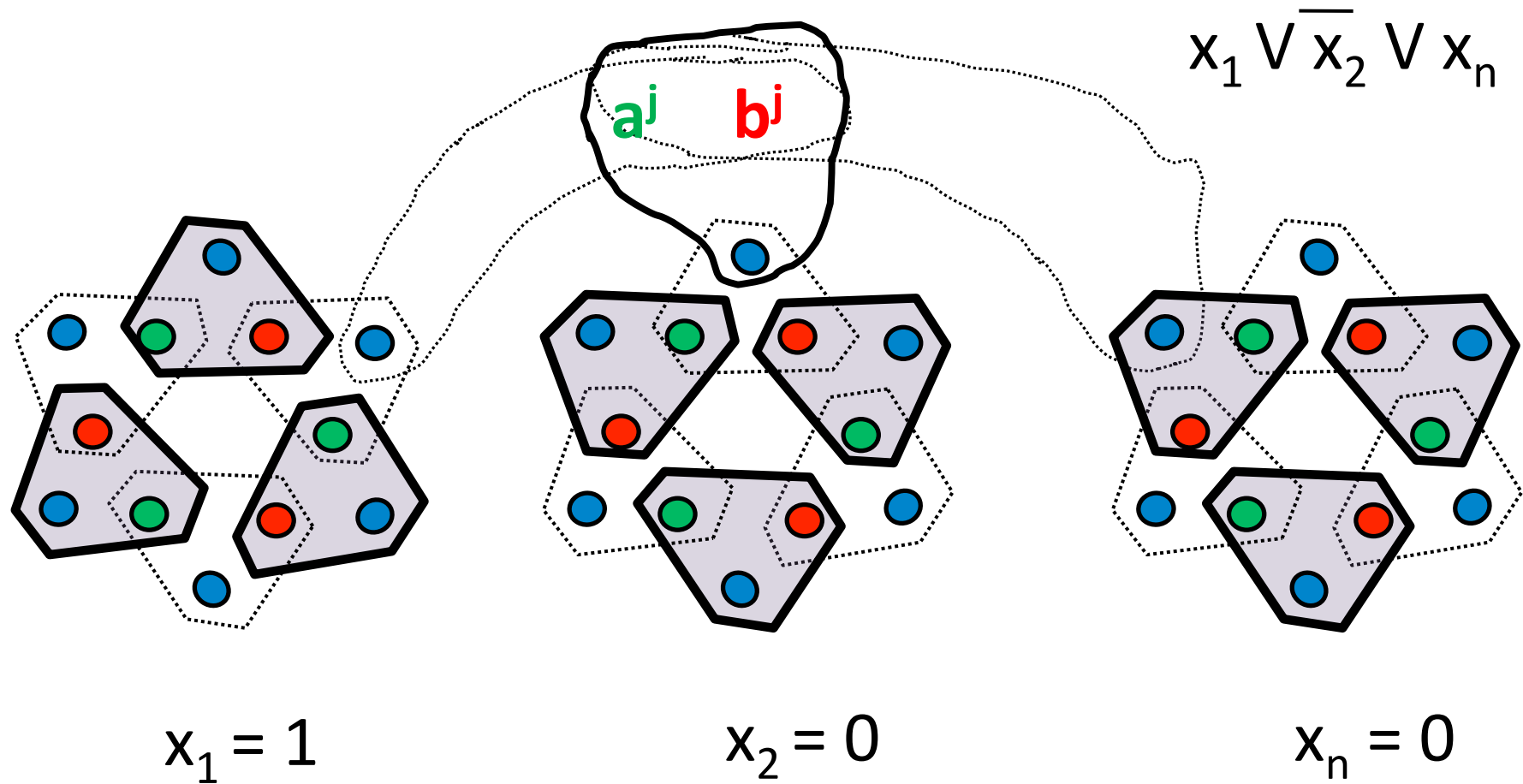Satisfying -> can choose a triple for each clause gadget.

Disjoint, and cover all of A and B.

$x_1 \lor \overline{x_2} \lor x_n$

a^j b^j

$x_1 = 1$ $x_2 = 0$ $x_n = 0$

Truth assignment -> choice of triples at variable gadgets.
Satisfying -> can choose a triple for each clause gadget.

Disjoint, and cover all of A and B.

$x_1 \vee \overline{x_2} \vee x_n$

$a^j \quad b^j$

$x_1 = 1 \qquad x_2 = 0 \qquad x_n = 0$

Cover all internals (A,B) once
        -> truth assignment (var gadgets)
Cover all clause internal elements -> satisfies clause

a^j b^j

$x_1 \vee \overline{x}_2 \vee x_n$

$x_1 = 1$ $x_2 = 0$ $x_n = 0$