Design and Analysis of Algorithms **out:** January 15, 2014

Problem Set 0

Lecturer: Daniel A. Spielman **due:** Never

This is just to test your background

I wrote these problems to help you decide if you know enough discrete math to take this course. You should be able to solve these problems after a little thought. I am not yet sure if I will write solutions to these.

Problem 1: Defining a Tree

A graph T with n vertices is called a tree if it satisfies any two of the following three properties:

1. T has no cycles.
2. T has $n - 1$ edges.
3. T is connected.

Prove that any of those two imply the third.

Problem 2: Re-defining a Tree

Here's another definition of a tree: a tree is a connected graph such that for every two vertices x and y, there is exactly one path in the graph between x and y. Prove that this definition is equivalent to the definition from Problem 1.

Problem 3: Another property of trees.

Let T be a tree. Prove that T contains a v vertex of degree 1. Let (u, v) be the unique edge involving v. Prove that if we remove v and edge (u, v) from T, then the graph that remains is also a tree.