

# Iterative (Turbo) Soft Interference Cancellation and Decoding for Coded CDMA

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**Abstract**—The presence of both multiple-access interference (MAI) and intersymbol interference (ISI) constitutes a major impediment to reliable communications in multipath code-division multiple-access (CDMA) channels. In this paper, an iterative receiver structure is proposed for decoding multiuser information data in a convolutionally coded asynchronous multipath DS-SS-CDMA system. The receiver performs two successive soft-output decisions, achieved by a soft-input soft-output (SISO) multiuser detector and a bank of single-user SISO channel decoders, through an iterative process. At each iteration, extrinsic information is extracted from detection and decoding stages and is then used as *a priori* information in the next iteration, just as in Turbo decoding. Given the multipath CDMA channel model, a direct implementation of a sliding-window SISO multiuser detector has a prohibitive computational complexity. A low-complexity SISO multiuser detector is developed based on a novel nonlinear interference suppression technique, which makes use of both soft interference cancellation and instantaneous linear minimum mean-square error filtering. The properties of such a nonlinear interference suppressor are examined, and an efficient recursive implementation is derived. Simulation results demonstrate that the proposed low-complexity iterative receiver structure for interference suppression and decoding offers significant performance gain over the traditional noniterative receiver structure. Moreover, at high signal-to-noise ratio, the detrimental effects of MAI and ISI in the channel can almost be completely overcome by iterative processing, and single-user performance can be approached.

**Index Terms**—Coded CDMA, instantaneous MMSE filtering, multiuser detection, soft interference cancellation, Turbo processing.

## I. INTRODUCTION

OVER THE PAST decade, a significant amount of research has addressed various multiuser detection methods for interference suppression in code-division multiple-access (CDMA) communication systems [19]. The high computational complexity of the optimal multiuser detectors (which is exponential in terms of the number of users in the channel) has motivated the study of a number of low-complexity suboptimal multiuser detectors. These low-complexity methods fall

largely into two categories: linear detectors and nonlinear detectors. A linear detector is comprised of a linear filter applied to the received signal, followed by a scalar quantizer. The nonlinear detectors are based primarily on various techniques for successive cancellation of interference.

Most of the previous work on multiuser detection focused on uncoded CDMA systems, i.e., on the demodulation of multiuser signals. Since in practice, most CDMA systems employ error control coding and interleaving, recent work in this area has addressed multiuser detection for coded CDMA systems. In [6], it is shown that the optimal decoding scheme for an asynchronous convolutionally coded CDMA system combines the trellises of both the asynchronous multiuser detector and the convolutional code, resulting in a prohibitive computational complexity  $O(2^{K\nu})$ , where  $K$  is the number of users in the channel, and  $\nu$  is the code constraint length. In [7], some low-complexity receivers which perform multiuser symbol detection and decoding either separately or jointly are studied.

Recently iterative (“Turbo”) processing techniques have received considerable attention followed by the discovery of the powerful Turbo codes [2, 3]. The so-called Turbo-principle can be successfully applied to many detection/decoding problems such as serial concatenated decoding, equalization, coded modulation, multiuser detection and joint source and channel decoding [9]. In particular, a Turbo equalization scheme is proposed in [4] for convolutionally coded digital transmission over intersymbol interference channel. More recently, in [11] an optimal iterative multiuser detector for synchronous coded CDMA system is derived, based on iterative techniques for cross-entropy minimization. A practical suboptimal implementation is also presented. The computational complexity of this method, however, is  $O(2^K + 2^\nu)$ , which is still prohibitive for channels with medium to large number of users. A similar work has also appeared in [16].

The presence of both multiple-access interference (MAI) and intersymbol interference (ISI) constitutes a major impediment to reliable CDMA communications in multipath channels. Linear techniques for joint multiuser detection and equalization have been proposed as effective ways for combating both types of interference [20]. The purpose of this paper is to develop low-complexity iterative multiuser receivers for coded CDMA systems over multipath channels. The rest of the paper is organized as follows. In Section II, the multipath CDMA signal model is presented, and the iterative (Turbo) receiver structure for multiuser detection and decoding is outlined. In Section III, a soft-input soft-output (SISO)

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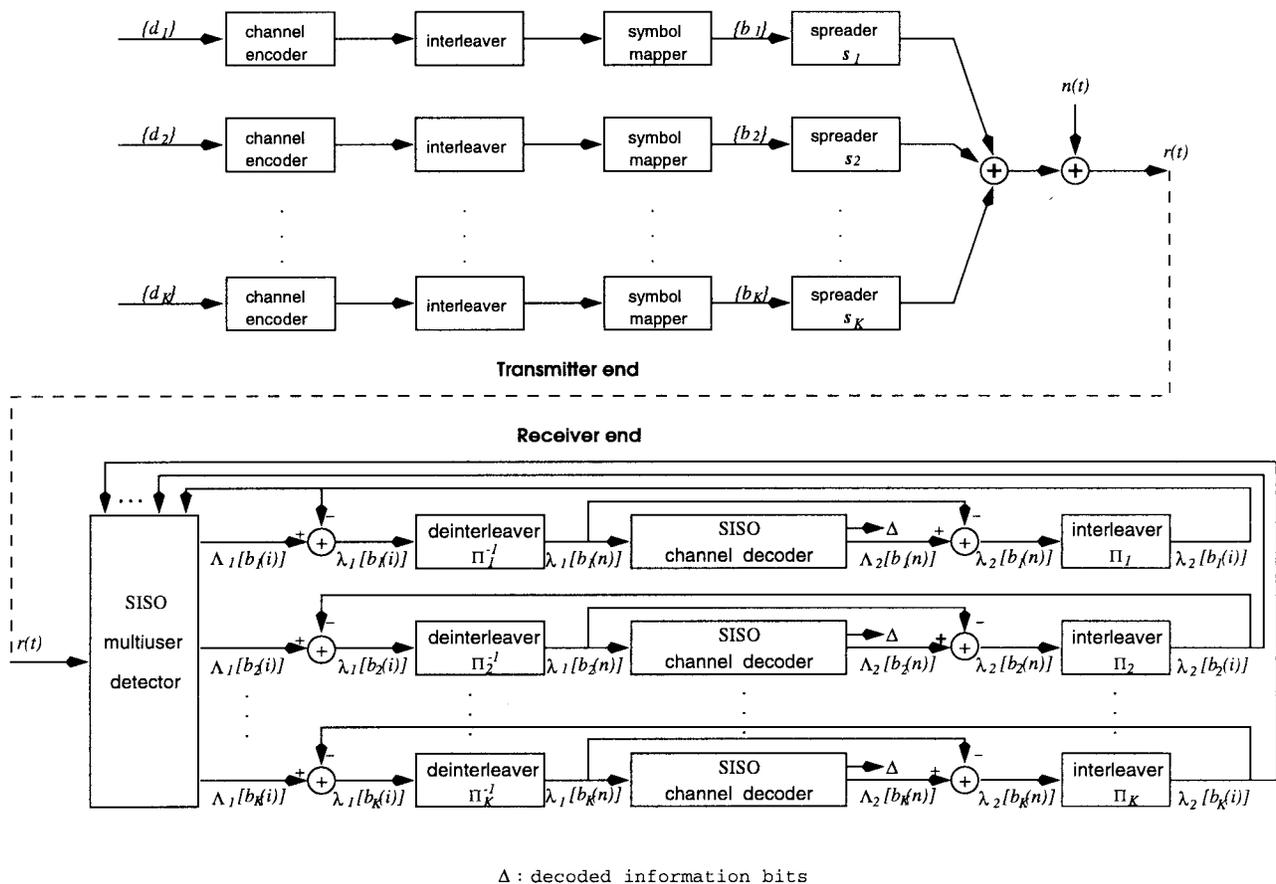


Fig. 1. A coded CDMA system with iterative (Turbo) multiuser receiver.

convolutional channel decoder is described. In Section IV, an exact SISO multiuser detector for synchronous CDMA channel is first derived; then a low-complexity approximate SISO multiuser detector is developed, which is based on soft interference cancellation and linear minimum mean-square error (MMSE) filtering. In Section V, the exact as well as a low-complexity approximate sliding-window SISO multiuser detectors are developed for multipath CDMA channels. Section VI contains the conclusions.

## II. SYSTEM DESCRIPTION

### A. Signal Model

We consider a convolutionally coded CDMA system with  $K$  users, employing normalized modulation waveforms  $s_1, s_2, \dots, s_K$ , and signaling through respective multipath channels with additive white Gaussian noise. The block diagram of the transmitter-end of such a system is shown in the upper half of Fig. 1. The binary information data  $\{d_k(m)\}$  for user  $k$ ,  $k = 1, \dots, K$ , are convolutionally encoded with code rate  $R_k$ . A code-bit interleaver is used to reduce the influence of the error bursts at the input of each channel decoder. The interleaved code-bits of the  $k$ th user are BPSK symbol mapped, yielding data symbols of duration  $T$ . Each data symbol  $b_k(i)$  is then modulated by a spreading waveform  $s_k(t)$ , and transmitted through the multipath channel.

The transmitted signal due to the  $k$ th user is given by

$$x_k(t) = A_k \sum_{i=0}^{M-1} b_k(i) s_k(t - iT) \quad (1)$$

where  $M$  is the number of data symbols per user per frame,  $T$  is the symbol interval, and  $A_k$  and  $\{s_k(t); 0 \leq t \leq T\}$  denote, respectively, the amplitude and normalized signaling waveform of the  $k$ th user. It is assumed that  $s_k(t)$  is supported only on the interval  $[0, T]$  and has unit energy. The  $k$ th user's signal  $x_k(t)$  propagates through a multipath channel with impulse response

$$g_k(t) = \sum_{l=0}^{L_k-1} g_{kl} \delta(t - \tau_{kl}) \quad (2)$$

where  $L_k$  is the number of paths in the  $k$ th user channel and  $g_{kl}$  and  $\tau_{kl}$  are, respectively, the complex gain and delay of the  $l$ th path of the  $k$ th user's signal. At the receiver, the received signal due to the  $k$ th user is then given by

$$\begin{aligned} y_k(t) &= x_k(t) \star g_k(t) \\ &= A_k \sum_{i=0}^{M-1} b_k(i) \sum_{l=0}^{L_k-1} g_{kl} s_k(t - iT - \tau_{kl}) \end{aligned} \quad (3)$$

where  $\star$  denotes convolution. The received signal at the receiver is the superposition of the  $K$  users' signals plus the

additive white Gaussian noise, given by

$$r(t) = \sum_{k=1}^K y_k(t) + \sigma n(t) \quad (4)$$

where  $n(t)$  is a zero-mean complex white Gaussian noise process with unit power spectral density, and  $\sigma^2$  is the variance of the ambient channel noise.

### B. Iterative (Turbo) Multiuser Receiver Structure

The iterative (Turbo) receiver structure is shown in the lower half of Fig. 1. It consists of two stages: a SISO multiuser detector, followed by  $K$  parallel single-user SISO channel decoders. The two stages are separated by deinterleavers and interleavers. The SISO multiuser detector delivers the *a posteriori* log-likelihood ratio (LLR) of a transmitted “+1” and a transmitted “-1” for every code bit of every user,

$$\Lambda_1[b_k(i)] \triangleq \log \frac{P[b_k(i) = +1 | r(t)]}{P[b_k(i) = -1 | r(t)]}, \quad k = 1, \dots, K; \quad i = 0, \dots, M-1. \quad (5)$$

Using Bayes' rule, (5) can be written as

$$\Lambda_1[b_k(i)] = \underbrace{\log \frac{p[r(t) | b_k(i) = +1]}{p[r(t) | b_k(i) = -1]}}_{\lambda_1[b_k(i)]} + \underbrace{\log \frac{P[b_k(i) = +1]}{P[b_k(i) = -1]}}_{\lambda_2^p[b_k(i)]}, \quad (6)$$

where the second term in (6), denoted by  $\lambda_2^p[b_k(i)]$ , represents the *a priori* LLR of the code bit  $b_k(i)$ , which is computed by the channel decoder of the  $k$ th user in the previous iteration, interleaved and then fed back to the SISO multiuser detector. (The superscript  $p$  indicates the quantity obtained from the previous iteration). For the first iteration, assuming equally likely code bits, i.e., no prior information available, we then have  $\lambda_2^p[b_k(i)] = 0$ , for  $1 \leq k \leq K$  and  $0 \leq i < M$ . The first term in (6), denoted by  $\lambda_1[b_k(i)]$ , represents the *extrinsic* information delivered by the SISO multiuser detector, based on the received signal  $r(t)$ , the structure of the multiuser signal given by (3) and (4), the prior information about the code bits of all other users,  $\{\lambda_2^p[b_l(j)], l \neq k, 0 \leq j < M\}$ , and the prior information about the code bits of the  $k$ th user other than the  $i$ th bit,  $\{\lambda_2^p[b_k(j)], j \neq i\}$ . The extrinsic information  $\lambda_1[b_k(i)]$ , which is not influenced by the *a priori* information  $\lambda_2^p[b_k(i)]$  provided by the channel decoder, is then reverse interleaved and fed into the  $k$ th user's channel decoder, as the *a priori* information in the next iteration.

Based on the prior information  $\lambda_1^p[b_k(n)]$ , and the trellis structure (i.e., code constraints) of the channel code, the  $k$ th user's SISO channel decoder computes the *a posteriori* LLR of each code bit

$$\begin{aligned} \Lambda_2[b_k(n)] &\triangleq \log \frac{P[b_k(n) = +1 | \{\lambda_1^p[b_k(n)]\}_{n=0}^{M-1}; \text{decoding}]}{P[b_k(n) = -1 | \{\lambda_1^p[b_k(n)]\}_{n=0}^{M-1}; \text{decoding}]} \\ &= \lambda_2[b_k(n)] + \lambda_1^p[b_k(n)], \\ &k = 1, \dots, K; \quad n = 0, \dots, M-1 \quad (7) \end{aligned}$$

where the second equality will be shown in the next section [cf. (16)]. It is seen from (7) that the output of the SISO channel decoder is the sum of the prior information  $\lambda_1^p[b_k(n)]$ , and the *extrinsic* information  $\lambda_2[b_k(n)]$  delivered by the channel decoder. As will be seen in the next section, this extrinsic information is the information about the code bit  $b_k(n)$  gleaned from the prior information about the other code bits,  $\{\lambda_1^p[b_k(m)]\}_{m \neq n}$ , based on the trellis constraint of the code. The SISO channel decoder also computes the *a posteriori* LLR of every information bit, which is used to make decision on the decoded bit at the last iteration. After interleaving, the extrinsic information delivered by the  $K$  channel decoders  $\{\lambda_2[b_k(i)]\}_{k=1}^K$  is then fed back to the SISO multiuser detector, as the prior information about the code bits of all users, in the next iteration. Note that at the first iteration, the extrinsic information  $\{\lambda_1[b_k(i)]\}$  and  $\{\lambda_2[b_k(i)]\}$  are statistically independent. But subsequently since they use the same information indirectly, they will become more and more correlated and finally the improvement through the iterations will diminish.

### III. SISO CHANNEL DECODER

The input to the  $k$ th SISO channel decoder are the *a priori* LLR's (or equivalently, probability distributions) of the code bits of the  $k$ th user. It delivers as output an update of the LLR's of the code bits, as well as the LLR's of the information bits, based on the code constraints. In this section, we outline a procedure for computing the LLR's of the information and code bits, which is essentially a slight modification of the algorithm in [1].

Consider a binary rate  $\frac{k_0}{n_0}$  convolutional encoder of overall constraint length  $k_0\nu$ . The input to the encoder at time  $t$  is the block  $\underline{d}_t = (d_t^1, \dots, d_t^{k_0})$  and the corresponding output is  $\underline{b}_t = (b_t^1, \dots, b_t^{n_0})$ . The state of the trellis at time  $t$  can be represented by a  $[k_0(\nu-1)]$ -tuple, as  $S_t = (s_t^1, \dots, s_t^{k_0(\nu-1)}) = (\underline{d}_{t-1}, \dots, \underline{d}_{t-\nu+1})$ . Denote the input information bits that cause the state transition from  $S_{t-1} = s'$  to  $S_t = s$  by  $\underline{d}(s', s)$  and the corresponding output code bits by  $\underline{b}(s', s)$ . Suppose that the encoder starts in state  $S_0 = \mathbf{0}$ . An information bit stream  $\{\underline{d}_t\}_{t=1}^T$ , are the input to the encoder, followed by  $\nu$  blocks of all zero inputs, causing the encoder to end in state  $S_\tau = \mathbf{0}$ , where  $\tau = T + \nu$ . Let  $\underline{b}_t$  denote the output of the channel encoder at time  $t$ . We use the notation

$$P[\underline{b}_t(s', s)] \triangleq P[\underline{b}_t = \underline{b}(s', s)]. \quad (8)$$

Define the forward and backward recursions as follows [1]

$$\alpha_t(s) = \sum_{s'} \alpha_{t-1}(s') P[\underline{b}_t(s', s)], \quad t = 1, 2, \dots, \tau \quad (9)$$

$$\beta_t(s) = \sum_{s'} \beta_{t+1}(s') P[\underline{b}_{t+1}(s, s')], \quad t = \tau-1, \tau-2, \dots, 0 \quad (10)$$

with boundary conditions  $\alpha_0(\mathbf{0}) = 1$ ,  $\alpha_0(s \neq \mathbf{0}) = 0$ ; and  $\beta_\tau(\mathbf{0}) = 1$ ,  $\beta_\tau(s \neq \mathbf{0}) = 0$ . In (9) the summation is over all the states  $s'$  where the transition  $(s', s)$  is possible. Similarly for the summation in (10).

A direct implementation of the recursions (9) and (10) is numerically unstable, since both  $\alpha_t(s)$  and  $\beta_t(s)$  drop toward zero exponentially. In order to obtain a numerically stable algorithm, these quantities must be scaled as the computation proceeds. Let  $\tilde{\alpha}_t(s)$  denote the scaled version of  $\alpha_t(s)$ . Initially,  $\alpha_1(s)$  is computed according to (9), and we set  $\hat{\alpha}_1(s) = \alpha_1(s)$ , and  $\tilde{\alpha}_1 = c_1 \hat{\alpha}_1(s)$ , with  $c_1 \triangleq 1/\sum_s \hat{\alpha}_1(s)$ . For each  $t \geq 2$ , we compute  $\tilde{\alpha}_t(s)$  according to

$$\hat{\alpha}_t(s) = \sum_{s'} \tilde{\alpha}_{t-1}(s') P[\underline{b}_t(s', s)] \quad (11)$$

$$\tilde{\alpha}_t(s) = c_t \hat{\alpha}_t(s), \quad \text{with } c_t = 1 / \sum_s \hat{\alpha}_t(s). \quad (12)$$

Now by a simple induction we obtain  $\tilde{\alpha}_{t-1}(s) = (\prod_{i=1}^{t-1} c_i) \alpha_{t-1}(s) \triangleq C_{t-1} \alpha_{t-1}(s)$ . Thus we can write  $\tilde{\alpha}_t(s)$  as

$$\tilde{\alpha}_t(s) = \frac{\sum_{s'} C_{t-1} \alpha_{t-1}(s') P[\underline{b}_t(s', s)]}{\sum_s \sum_{s'} C_{t-1} \alpha_{t-1}(s') P[\underline{b}_t(s', s)]} = \frac{\alpha_t(s)}{\sum_s \alpha_t(s)}. \quad (13)$$

That is, each  $\alpha_t(s)$  is effectively scaled by the sum over all states of  $\alpha_t(s)$ .

Let  $\tilde{\beta}_t(s)$  denote the scaled version of  $\beta_t(s)$ . Initially,  $\beta_{\tau-1}(s)$  is computed according to (10), and we set  $\hat{\beta}_{\tau-1}(s) = \beta_{\tau-1}(s)$ . For each  $t < \tau - 1$ , we compute  $\tilde{\beta}_t(s)$  according to

$$\hat{\beta}_t(s) = \sum_{s'} \tilde{\beta}_{t+1}(s') P[\underline{b}_{t+1}(s, s')] \quad (14)$$

$$\tilde{\beta}_t(s) = c_t \hat{\beta}_t(s). \quad (15)$$

By induction, we can write  $\tilde{\beta}_t(s)$  as  $\tilde{\beta}_t(s) = (\prod_{i=t}^{\tau} c_i) \beta_t(s) \triangleq D_t \beta_t(s)$ . Let  $\mathcal{S}_j^+$  be the set of state pairs  $(s', s)$  such that the  $j$ th bit of the code symbol  $\underline{b}(s', s)$  is +1. Similarly, define  $\mathcal{S}_j^-$ . The *a posteriori* LLR of the code bit  $b_t^j$  at the output of the channel decoder is given by

$$\begin{aligned} \Lambda_2[b_t^j] &\triangleq \log \frac{P[b_t^j = +1 \mid \text{decoding}]}{P[b_t^j = -1 \mid \text{decoding}]} \\ &= \log \frac{\sum_{\mathcal{S}_j^+} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i=1}^{n_0} P[b_t^i(s', s)]}{\sum_{\mathcal{S}_j^-} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i=1}^{n_0} P[b_t^i(s', s)]} \\ &= \log \frac{\sum_{\mathcal{S}_j^+} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i \neq j} P[b_t^i(s', s)]}{\sum_{\mathcal{S}_j^-} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i \neq j} P[b_t^i(s', s)]} \\ &\quad + \log \frac{P[b_t^j = +1]}{P[b_t^j = -1]} \\ &= \log \underbrace{\frac{\sum_{\mathcal{S}_j^+} \tilde{\alpha}_{t-1}(s') \cdot \tilde{\beta}_t(s) \cdot \prod_{i \neq j} P[b_t^i(s', s)]}{\sum_{\mathcal{S}_j^-} \tilde{\alpha}_{t-1}(s') \cdot \tilde{\beta}_t(s) \cdot \prod_{i \neq j} P[b_t^i(s', s)]}}_{\lambda_2[b_t^j]} \\ &\quad + \log \underbrace{\frac{P[b_t^j = +1]}{P[b_t^j = -1]}}_{\lambda_1^p[b_t^j]} \end{aligned} \quad (16)$$

where the last equality follows from the fact that  $C_{t-1} D_t = \prod_{i=1}^{t-1} c_i \cdot \prod_{i=t}^{\tau} c_i = \prod_{i=1}^{\tau} c_i$  is a constant which is independent

of  $t$ . It is seen from (16) that the output of the SISO channel decoder is the sum of the prior information  $\lambda_1^p[b_t^j]$  provided by the SISO multiuser detector, and the extrinsic information  $\lambda_2[b_t^j]$ . The extrinsic information is the information about the code bit  $b_t^j$  gleaned from the prior information about the other code bits based on the trellis structure of the code.

We can also compute the *a posteriori* LLR of the information symbol bit. Let  $\mathcal{U}_j^+$  be the set of state pairs  $(s', s)$  such that the  $j$ th bit of the information symbol  $\underline{d}(s', s)$  is +1. Similarly define  $\mathcal{U}_j^-$ . Then we have

$$\Lambda_2[d_t^j] = \log \frac{\sum_{\mathcal{U}_j^+} \tilde{\alpha}_{t-1}(s') \cdot \tilde{\beta}_t(s) \cdot \prod_{i=1}^{n_0} P[b_t^i(s', s)]}{\sum_{\mathcal{U}_j^-} \tilde{\alpha}_{t-1}(s') \cdot \tilde{\beta}_t(s) \cdot \prod_{i=1}^{n_0} P[b_t^i(s', s)]}. \quad (17)$$

Note that the LLR's of the information bits are only computed at the last iteration. The information bit  $d_t^j$  is then decoded according to  $\hat{d}_t^j = \text{sgn}(\Lambda_2[d_t^j])$ .

Finally, since the input to the SISO channel decoder is the LLR of the code bits,  $\lambda_1^p[b_t^i]$ , as will be shown in the next section, the code bit distribution  $P[b_t^i(s', s)]$  can be expressed in terms of its LLR as [cf. (27)]

$$P[b_t^i(s', s)] = \frac{1}{2} \left[ 1 + b^i(s', s) \tanh \left( \frac{1}{2} \lambda_1^p[b_t^i] \right) \right]. \quad (18)$$

#### IV. SISO MULTIUSER DETECTORS FOR SYNCHRONOUS CDMA

In this section, we focus on a special case of the channel model (2), where  $g_k(t) = \delta(t)$ , for  $k = 1, \dots, K$ , i.e., the synchronous CDMA system. The received (real-valued) signal can then be written as

$$r(t) = \sum_{k=1}^K A_k \sum_{i=0}^{M-1} b_k(i) s_k(t - iT) + \sigma n(t) \quad (19)$$

where  $n(t)$  is a zero-mean white Gaussian noise process with unit power spectral density. For this synchronous case, it is easily seen that a sufficient statistic for demodulating the  $i$ th code bits of the  $K$  users is given by the  $K$ -vector  $\underline{y}(i)$  whose  $k$ th component is the output of a filter matched to  $s_k$  in the  $i$ th code bit interval, i.e.,

$$y_k(i) \triangleq \int_{iT}^{(i+1)T} s_k(t - iT) r(t) dt, \quad k = 1, \dots, K. \quad (20)$$

This sufficient vector  $\underline{y}(i)$  can be written as [19]

$$\underline{y}(i) = \underline{R} \underline{A} \underline{b}(i) + \sigma \underline{n}(i) \quad (21)$$

where  $\underline{R}$  denotes the normalized cross-correlation matrix of the signal set  $s_1, \dots, s_K$ :  $[\underline{R}]_{k,l} = \rho_{kl} \triangleq \int_0^T s_k(t) s_l(t) dt$ ;  $\underline{A} \triangleq \text{diag}(A_1, \dots, A_K)$ ;  $\underline{b}(i) = [b_1(i), \dots, b_K(i)]^T$ ; and  $\underline{n}(i) \sim \mathcal{N}(\underline{0}, \underline{R})$  is a Gaussian noise vector, independent of  $\underline{b}(i)$ . In what follows we derive the exact as well as a low-complexity approximate SISO multiuser detector as the first stage of the Turbo multiuser receiver for the synchronous CDMA channel. The key techniques developed in this section can be generalized and applied to the more general multipath CDMA channel, as will be discussed in Section V.

### A. SISO Multiuser Detector

Denote

$$\mathcal{B}_k^+ \triangleq \{(b_1, \dots, b_{k-1}, +1, b_{k+1}, \dots, b_K) : b_j \in \{+1, -1\}, j \neq k\}. \quad (22)$$

Similarly define  $\mathcal{B}_k^-$ . From (21), the extrinsic information  $\lambda_1[b_k(i)]$  delivered by the SISO multiuser detector [cf. (6)] is shown in (23), at the bottom of the page, where we use the notation  $P[b_j] \triangleq P[b_j(i) = b_j]$ . The summations in the numerator (resp., denominator) in (23) are over all the  $2^{K-1}$  possible vectors  $\underline{b}$  in  $\mathcal{B}_k^+$  (resp.,  $\mathcal{B}_k^-$ ). We have

$$\begin{aligned} & \exp[-(\underline{y}(i) - \underline{R}\underline{A}\underline{b})^T \underline{R}^{-1}(\underline{y}(i) - \underline{R}\underline{A}\underline{b})/(2\sigma^2)] \\ &= \exp[-\underline{y}(i)^T \underline{R}^{-1} \underline{y}(i)/(2\sigma^2)] \exp[-\underline{b}^T \underline{A}\underline{R}\underline{A}\underline{b}/(2\sigma^2)] \\ & \quad \times \exp[\underline{b}^T \underline{A}\underline{y}(i)/\sigma^2]. \end{aligned} \quad (24)$$

Note that the first term in (24) is independent of  $\underline{b}$  and therefore will be cancelled in (23). The third term in (24) can be written

$$\begin{aligned} & \exp[\underline{b}^T \underline{A}\underline{y}(i)/\sigma^2] \\ &= \exp\left[\sum_{j=1}^K A_j y_j(i) b_j / \sigma^2\right] \\ &= \prod_{j=1}^K \exp[A_j y_j(i) b_j / \sigma^2] \\ &= \prod_{j=1}^K \left[ \frac{1+b_j}{2} \exp\left(A_j y_j(i) / \sigma^2\right) \right. \\ & \quad \left. + \frac{1-b_j}{2} \exp\left(-A_j y_j(i) / \sigma^2\right) \right] \quad (25) \\ &= \prod_{j=1}^K \left\{ \frac{1}{2} \left[ \exp\left(A_j y_j(i) / \sigma^2\right) + \exp\left(-A_j y_j(i) / \sigma^2\right) \right] \right. \\ & \quad \left. + \frac{b_j}{2} \left[ \exp\left(A_j y_j(i) / \sigma^2\right) - \exp\left(-A_j y_j(i) / \sigma^2\right) \right] \right\} \\ &= \prod_{j=1}^K \left[ \cosh\left(A_j y_j(i) / \sigma^2\right) \right] \left[ 1 + b_j \tanh(A_j y_j(i) / \sigma^2) \right] \end{aligned} \quad (26)$$

where (25) follows from the fact that  $b_j \in \{+1, -1\}$ . The first term in (26) is also independent of  $\underline{b}$  and will be cancelled in (23). In (23) the *a priori* probabilities of the code bits can be expressed in terms of their LLR's  $\lambda_2^p[b_j(i)]$ , as follows. Since  $\lambda_2^p[b_j(i)] = \log \frac{P[b_j(i)=+1]}{P[b_j(i)=-1]}$ , after some manipulations, we have for  $b_j \in \{+1, -1\}$ ,

$$\begin{aligned} P[b_j] &\triangleq P[b_j(i) = b_j] \\ &= \frac{\exp(b_j \lambda_2^p[b_j(i)])}{1 + \exp(b_j \lambda_2^p[b_j(i)])} \\ &= \frac{\exp(\frac{1}{2} b_j \lambda_2^p[b_j(i)])}{\exp(-\frac{1}{2} b_j \lambda_2^p[b_j(i)]) + \exp(\frac{1}{2} b_j \lambda_2^p[b_j(i)])} \\ &= \frac{\cosh(\frac{1}{2} \lambda_2^p[b_j(i)]) [1 + b_j \tanh(\frac{1}{2} \lambda_2^p[b_j(i)])]}{2 \cosh(\frac{1}{2} \lambda_2^p[b_j(i)])} \\ &= \frac{1}{2} \left[ 1 + b_j \tanh\left(\frac{1}{2} \lambda_2^p[b_j(i)\right) \right] \end{aligned} \quad (27)$$

where (27) follows from a similar derivation as that of (26). Substituting (24), (26) and (27) into (23) we obtain (28), shown at the bottom of the page. It is seen from (28) that the extrinsic information  $\lambda_1[b_k(i)]$  at the output of the SISO multiuser detector consists of two parts, the first term is the channel value of the desired user  $A_k y_k(i) / \sigma^2$ , and the second term is the information extracted from the other users' channel values  $\{y_j(i)\}_{j \neq k}$  as well as their prior information  $\{\lambda_2^p[b_j(i)]\}_{j \neq k}$ .

### B. Low-Complexity SISO Multiuser Detector

It is clear from (28) that the computational complexity of the SISO multiuser detector is exponential in terms of the number of users  $K$ , which is certainly prohibitive for channels with medium to large number of users. In what follows we develop a low-complexity approximate SISO multiuser detector based on soft interference cancellation and linear MMSE filtering.

1) *Soft Instantaneous MMSE Interference Cancellation/Suppression* Based on the *a priori* LLR of the code bits of all users,  $\{\lambda_2^p[b_j(i)]\}_{j=1}^K$ , provided by the SISO channel decoder from the previous stage, we first form soft estimates

$$\lambda_1[b_k(i)] = \log \frac{\sum_{\underline{b} \in \mathcal{B}_k^+} \exp\left[-(\underline{y}(i) - \underline{R}\underline{A}\underline{b})^T \underline{R}^{-1}(\underline{y}(i) - \underline{R}\underline{A}\underline{b})/(2\sigma^2)\right] \prod_{j \neq k} P[b_j]}{\sum_{\underline{b} \in \mathcal{B}_k^-} \exp\left[-(\underline{y}(i) - \underline{R}\underline{A}\underline{b})^T \underline{R}^{-1}(\underline{y}(i) - \underline{R}\underline{A}\underline{b})/(2\sigma^2)\right] \prod_{j \neq k} P[b_j]} \quad (23)$$

$$\lambda_1[b_k(i)] = \frac{2A_k y_k(i)}{\sigma^2} + \log \frac{\sum_{\underline{b} \in \mathcal{B}_k^+} \left\{ \exp\left[-\underline{b}^T \underline{A}\underline{R}\underline{A}\underline{b}/(2\sigma^2)\right] \prod_{j \neq k} [1 + b_j \tanh(A_j y_j(i) / \sigma^2)] \left[ 1 + b_j \tanh\left(\frac{1}{2} \lambda_2^p[b_j(i)\right) \right] \right\}}{\sum_{\underline{b} \in \mathcal{B}_k^-} \left\{ \exp\left[-\underline{b}^T \underline{A}\underline{R}\underline{A}\underline{b}/(2\sigma^2)\right] \prod_{j \neq k} [1 + b_j \tanh(A_j y_j(i) / \sigma^2)] \left[ 1 + b_j \tanh\left(\frac{1}{2} \lambda_2^p[b_j(i)\right) \right] \right\}} \quad (28)$$

of the code bits of all users as

$$\begin{aligned}\tilde{b}_j(i) &\triangleq \sum_{b_j \in \{+1, -1\}} b_j P[b_j] \\ &= \sum_{b_j \in \{+1, -1\}} \frac{b_j}{2} \left[ 1 + b_j \tanh\left(\frac{1}{2} \lambda_2^p [b_j(i)]\right) \right] \\ &= \tanh\left(\frac{1}{2} \lambda_2^p [b_j(i)]\right), \quad j = 1, \dots, K\end{aligned}\quad (29)$$

where the second equality follows from (27). Define

$$\begin{aligned}\tilde{\mathbf{b}}(i) &\triangleq [\tilde{b}_1(i) \quad \dots \quad \tilde{b}_K(i)], \\ \tilde{\mathbf{b}}_k(i) &\triangleq \tilde{\mathbf{b}}(i) - \tilde{b}_k(i) \mathbf{e}_k \\ &= [\tilde{b}_1(i), \dots, \tilde{b}_{k-1}(i), 0, \tilde{b}_{k+1}(i), \dots, \tilde{b}_K(i)]^T,\end{aligned}\quad (30)$$

where  $\mathbf{e}_k$  denotes a  $K$ -vector of all zeros, except for the  $k$ th element, which is 1. Therefore,  $\tilde{\mathbf{b}}_k(i)$  is obtained from  $\tilde{\mathbf{b}}(i)$  by setting the  $k$ th element to zero. For each user  $k$ , a soft interference cancellation is performed on the matched-filter output  $\underline{\mathbf{y}}(i)$  in (21), to obtain

$$\underline{\mathbf{y}}_k(i) \triangleq \underline{\mathbf{y}}(i) - \underline{\mathbf{R}} \underline{\mathbf{A}} \tilde{\mathbf{b}}_k(i) = \underline{\mathbf{R}} \underline{\mathbf{A}} [b(i) - \tilde{b}_k(i)] + \sigma \mathbf{n}(i), \quad k = 1, \dots, K. \quad (32)$$

Such a soft interference cancellation scheme was first proposed in [8]. Next, in order to further suppress the residual interference in  $\underline{\mathbf{y}}_k(i)$ , an instantaneous linear MMSE filter  $\underline{\mathbf{w}}_k(i)$  is applied to  $\underline{\mathbf{y}}_k(i)$ , to obtain

$$z_k(i) = \underline{\mathbf{w}}_k(i)^T \underline{\mathbf{y}}_k(i), \quad (33)$$

where the filter  $\underline{\mathbf{w}}_k(i) \in \mathcal{R}^K$  is chosen to minimize the mean-square error between the code bit  $b_k(i)$  and the filter output  $z_k(i)$ , i.e.,

$$\begin{aligned}\underline{\mathbf{w}}_k(i) &= \arg \min_{\underline{\mathbf{w}} \in \mathcal{R}^K} E \left\{ \left[ b_k(i) - \underline{\mathbf{w}}^T \underline{\mathbf{y}}_k(i) \right]^2 \right\} \\ &= \arg \min_{\underline{\mathbf{w}} \in \mathcal{R}^K} \underline{\mathbf{w}}^T E \left\{ \underline{\mathbf{y}}_k(i) \underline{\mathbf{y}}_k(i)^T \right\} \underline{\mathbf{w}} \\ &\quad - 2 \underline{\mathbf{w}}^T E \left\{ b_k(i) \underline{\mathbf{y}}_k(i) \right\}\end{aligned}\quad (34)$$

where using (32), we have

$$E \left\{ \underline{\mathbf{y}}_k(i) \underline{\mathbf{y}}_k(i)^T \right\} = \underline{\mathbf{R}} \underline{\mathbf{A}} \text{cov} \left\{ b(i) - \tilde{b}_k(i) \right\} \underline{\mathbf{A}} \underline{\mathbf{R}} + \sigma^2 \underline{\mathbf{R}} \quad (35)$$

$$\begin{aligned}E \left\{ b_k(i) \underline{\mathbf{y}}_k(i) \right\} &= \underline{\mathbf{R}} \underline{\mathbf{A}} E \left\{ b_k(i) \left[ b(i) - \tilde{b}_k(i) \right] \right\} \\ &= \underline{\mathbf{R}} \underline{\mathbf{A}} \mathbf{e}_k\end{aligned}\quad (36)$$

and in (35)

$$\begin{aligned}\text{cov} \left\{ b(i) - \tilde{b}_k(i) \right\} &= \text{diag} \left[ \text{var} \{ b_1(i) \}, \dots, \text{var} \{ b_{k-1}(i) \}, 1, \right. \\ &\quad \left. \text{var} \{ b_{k+1}(i) \}, \dots, \text{var} \{ b_K(i) \} \right] \\ &= \text{diag} \left[ 1 - \tilde{b}_1(i)^2, \dots, 1 - \tilde{b}_{k-1}(i)^2, 1, \right. \\ &\quad \left. 1 - \tilde{b}_{k+1}(i)^2, \dots, 1 - \tilde{b}_K(i)^2 \right]\end{aligned}\quad (37)$$

because

$$\text{var} \{ b_j(i) \} = E \{ b_j(i)^2 \} - [E \{ b_j(i) \}]^2 = 1 - \tilde{b}_j(i)^2. \quad (38)$$

Denote

$$\begin{aligned}\underline{\mathbf{V}}_k(i) &\triangleq \underline{\mathbf{A}} \text{cov} \left\{ b(i) - \tilde{b}_k(i) \right\} \underline{\mathbf{A}} \\ &= \sum_{j \neq k} A_j^2 [1 - \tilde{b}_j(i)^2] \mathbf{e}_j \mathbf{e}_j^T + A_k^2 \mathbf{e}_k \mathbf{e}_k^T.\end{aligned}\quad (39)$$

Substituting (35) and (36) into (34) we get

$$\begin{aligned}\underline{\mathbf{w}}_k(i) &= [\underline{\mathbf{R}} \underline{\mathbf{V}}_k(i) \underline{\mathbf{R}} + \sigma^2 \underline{\mathbf{R}}]^{-1} \underline{\mathbf{R}} \underline{\mathbf{A}} \mathbf{e}_k \\ &= A_k \underline{\mathbf{R}}^{-1} [\underline{\mathbf{V}}_k(i) + \sigma^2 \underline{\mathbf{R}}^{-1}]^{-1} \mathbf{e}_k.\end{aligned}\quad (40)$$

Substituting (32) and (40) into (33), we obtain

$$z_k(i) = A_k \mathbf{e}_k^T [\underline{\mathbf{V}}_k(i) + \sigma^2 \underline{\mathbf{R}}^{-1}]^{-1} [\underline{\mathbf{R}}^{-1} \underline{\mathbf{y}}(i) - \underline{\mathbf{A}} \tilde{\mathbf{b}}_k(i)]. \quad (41)$$

Notice that the term  $\underline{\mathbf{R}}^{-1} \underline{\mathbf{y}}(i)$  in (41) is the output of a decorrelating multiuser filter. Next we consider some special cases of the output  $z_k(i)$ .

- 1) No prior information on the code bits of the interfering users: i.e.,  $\lambda_2^p [b_j(i)] = 0$ . In this case,  $\tilde{b}_k(i) = \underline{0}$ , and  $\underline{\mathbf{V}}_k(i) = \underline{\mathbf{A}}^2$ . Then (41) becomes

$$z_k(i) = A_k \mathbf{e}_k^T (\underline{\mathbf{R}} + \sigma^2 \underline{\mathbf{A}}^{-2})^{-1} \underline{\mathbf{y}}(i) \quad (42)$$

which is simply the output of the linear MMSE multiuser detector for user  $k$ .

- 2) Perfect prior information on the code bits of the interfering users, i.e.,  $\lambda_2^p [b_j(i)] = \pm \infty$ . In this case,  $\tilde{b}_k(i) = [b_1(i), \dots, b_{k-1}(i), 0, b_{k+1}(i), \dots, b_K(i)]$ , and  $\underline{\mathbf{V}}_k(i) = A_k^2 \mathbf{e}_k \mathbf{e}_k^T$ . Substituting this result into (40), we obtain

$$\begin{aligned}\underline{\mathbf{w}}_k(i) &= A_k \underline{\mathbf{R}}^{-1} \left( A_k^2 \mathbf{e}_k \mathbf{e}_k^T + \sigma^2 \underline{\mathbf{R}} \right)^{-1} \mathbf{e}_k \\ &= \frac{A_k}{A_k^2 + \sigma^2} \mathbf{e}_k.\end{aligned}\quad (43)$$

The output of the soft instantaneous MMSE filter is then given by

$$\begin{aligned}z_k(i) &= \underline{\mathbf{w}}_k(i)^T \underline{\mathbf{y}}_k(i) \\ &= \frac{A_k}{A_k^2 + \sigma^2} \mathbf{e}_k^T \underline{\mathbf{y}}_k(i) \\ &= \frac{A_k}{A_k^2 + \sigma^2} \left[ y_k(i) - \sum_{j \neq k} A_j \rho_{kj} b_j(i) \right].\end{aligned}\quad (44)$$

That is, in this case, the output of the soft instantaneous MMSE filter is a scaled version of the  $k$ th user's matched filter output after ideal interference cancellation.

- 3) In general, the prior information provided by the SISO channel decoder satisfies  $0 < |\lambda_2^p[b_j(i)]| < \infty$ . The signal-to-interference-plus-noise ratio (SINR) at the soft instantaneous MMSE filter output is defined as

$$\text{SINR}[z_k(i)] \triangleq \frac{E^2\{z_k(i)\}}{\text{var}\{z_k(i)\}}. \quad (45)$$

Denote  $\underline{\text{SINR}}[z_k(i)]$  as the output SINR when there is no prior information on the code bits of interfering users, i.e., the SINR of the linear MMSE detector. Denote also  $\overline{\text{SINR}}[z_k(i)]$  as the output SINR when there is perfect prior information on the code bits of interfering users, i.e., the input signal-to-noise ratio (SNR) for the  $k$ th user, then it is shown in Appendix A that, if  $0 < |\lambda_2^p[b_j(i)]| < \infty$ , for  $1 \leq j \leq K$ , then we have

$$\overline{\text{SINR}}[z_k(i)] > \text{SINR}[z_k(i)] > \underline{\text{SINR}}[z_k(i)]. \quad (46)$$

Note that iterative soft interference cancellation schemes for uncoded and coded CDMA systems are proposed in [13] and [16], respectively, where no MMSE filtering across the users is performed after the cancellation stage. The instantaneous MMSE filtering proposed here provides an efficient and accurate way of computing the extrinsic information (as discussed next), which is vital to the Turbo multiuser receiver.

2) *Gaussian Approximation of Soft MMSE Filter Output* It is shown in [15] that the distribution of the residual interference-plus-noise at the output of a linear MMSE multiuser detector is well approximated by a Gaussian distribution. In what follows, we assume that the output of the soft instantaneous MMSE filter  $z_k(i)$  in (33) represents the output of an equivalent additive white Gaussian noise channel having  $b_k(i)$  as its input symbol. This equivalent channel can be represented as

$$z_k(i) = \mu_k(i)b_k(i) + \eta_k(i) \quad (47)$$

where  $\mu_k(i)$  is the equivalent amplitude of the  $k$ th user's signal at the output, and  $\eta_k(i) \sim \mathcal{N}(0, \nu_k^2(i))$  is a Gaussian noise sample. Using (32) and (33), the parameters  $\mu_k(i)$  and  $\nu_k^2(i)$  can be computed as follows, where the expectation is taken with respect to the code bits of interfering users  $\{b_j(i)\}_{j \neq k}$  and the channel noise vector  $\underline{n}(i)$ .

$$\begin{aligned} \mu_k(i) &= E\{z_k(i)b_k(i)\} = A_k \underline{e}_k^T \left[ \underline{V}_k(i) + \sigma^2 \underline{R}^{-1} \right]^{-1} \\ &\quad \times E \left\{ b_k(i) \underline{A} \left[ \underline{b}(i) - \tilde{\underline{b}}_k(i) \right] + b_k(i) \sigma \underline{n}(i) \right\} \\ &= A_k^2 \underline{e}_k^T \left[ \underline{V}_k(i) + \sigma^2 \underline{R}^{-1} \right]^{-1} \underline{e}_k \\ &= A_k^2 \left[ \underline{V}_k(i) + \sigma^2 \underline{R}^{-1} \right]^{-1}_{kk} \end{aligned} \quad (48)$$

$$\begin{aligned} \nu_k^2(i) &= \text{var}\{z_k(i)\} = E\{z_k(i)^2\} - \mu_k(i)^2 \\ &= \underline{w}_k(i) E \{ \underline{y}_k(i) \underline{y}_k(i)^T \} \underline{w}_k(i) - \mu_k(i)^2 \\ &= A_k^2 \underline{e}_k^T \left[ \underline{V}_k(i) + \sigma^2 \underline{R}^{-1} \right]^{-1} \underline{e}_k - \mu_k(i)^2 \\ &= \mu_k(i) - \mu_k(i)^2. \end{aligned} \quad (49)$$

From (47) the extrinsic information delivered by the soft instantaneous MMSE filter is then

$$\begin{aligned} \lambda_1[b_k(i)] &\triangleq \log \frac{p[z_k(i) | b_k(i) = +1]}{p[z_k(i) | b_k(i) = -1]} \\ &= -\frac{[z_k(i) - \mu_k(i)]^2}{2\nu_k^2(i)} + \frac{[z_k(i) + \mu_k(i)]^2}{2\nu_k^2(i)} \\ &= \frac{2\mu_k(i)z_k(i)}{\nu_k^2(i)} = \frac{2z_k(i)}{1 - \mu_k(i)}. \end{aligned} \quad (50)$$

3) *Recursive Procedure for Computing Soft Output* It is seen from (50) that in order to form the extrinsic LLR  $\lambda_1[b_k(i)]$  at the soft instantaneous MMSE filter, we must first compute  $z_k(i)$  and  $\mu_k(i)$ . From (33) and (48) the computation of  $z_k(i)$  and  $\mu_k(i)$  involves inverting a  $K \times K$  matrix, i.e.,

$$\underline{\Phi}_k(i) \triangleq \left[ \underline{V}_k(i) + \sigma^2 \underline{R}^{-1} \right]^{-1}. \quad (51)$$

Next we outline a recursive procedure for computing  $\underline{\Phi}_k$ . Define  $\underline{\Psi}^{(0)} \triangleq \sigma^2 \underline{R}$ , and

$$\underline{\Psi}^{(k)} \triangleq \left( \sigma^2 \underline{R}^{-1} + \sum_{j=1}^k A_j^2 [1 - \tilde{b}_j(i)^2] \underline{e}_j \underline{e}_j^T \right)^{-1}, \quad k = 1, \dots, K. \quad (52)$$

Using the matrix inversion lemma,  $\underline{\Psi}^{(k)}$  can be computed recursively as for  $k = 1, \dots, K$

$$\begin{aligned} \underline{\Psi}^{(k)} &= \underline{\Psi}^{(k-1)} - \frac{1}{A_k^{-2} [1 - \tilde{b}_k(i)^2]^{-1} + [\underline{\Psi}^{(k-1)}]_{kk}} \\ &\quad \times \left[ \underline{\Psi}^{(k-1)} \underline{e}_k \right] \left[ \underline{\Psi}^{(k-1)} \underline{e}_k \right]^T \end{aligned} \quad (53)$$

Denote  $\underline{\Psi} \triangleq \underline{\Psi}^{(K)}$ . Using the definition of  $\underline{V}_k(i)$  given by (39), we can then compute  $\underline{\Phi}_k$  from  $\underline{\Psi}$  as follows. For  $k = 1, \dots, K$

$$\begin{aligned} \underline{\Phi}_k(i) &= \left[ \underline{\Psi}^{-1} + A_k^2 \tilde{b}_k(i)^2 \underline{e}_k \underline{e}_k^T \right]^{-1} \\ &= \underline{\Psi} - \frac{1}{[A_k \tilde{b}_k(i)]^{-2} + [\underline{\Psi}]_{kk}} [\underline{\Psi} \underline{e}_k] [\underline{\Psi} \underline{e}_k]^T. \end{aligned} \quad (54)$$

Next we examine the computational complexity of the approximate SISO multiuser detector discussed in this section. From the above discussion, it is seen that at each symbol time  $i$ , the dominant computation involved in computing the matrix  $\underline{\Phi}_k(i)$ , for  $k = 1, \dots, K$ , consists of  $(2K) K$ -vector outer products, i.e.,  $K$  outer products in computing  $\underline{\Psi}^{(k)}$  as in (53), and  $K$  outer products in computing  $\underline{\Phi}_k(i)$  as in (54). From (48) and (50), in order to obtain the soft output  $\lambda_1[b_k(i)]$ , we also need to compute the soft instantaneous MMSE filter output  $z_k(i)$ , which by (41), is dominated by two  $K$ -vector inner products, i.e., one in computing the  $k$ th user's decorrelating filter output, and another in computing the final  $z_k(i)$ . Therefore, in computing the soft output of the approximate SISO multiuser detector, the dominant computation *per user per symbol* involves two  $K$ -vector outer products and two  $K$ -vector inner products. The total computational complexity of this Turbo multiuser detector is then  $O(K^2 + 2\nu)$ , where  $\nu$  is the code constraint length.

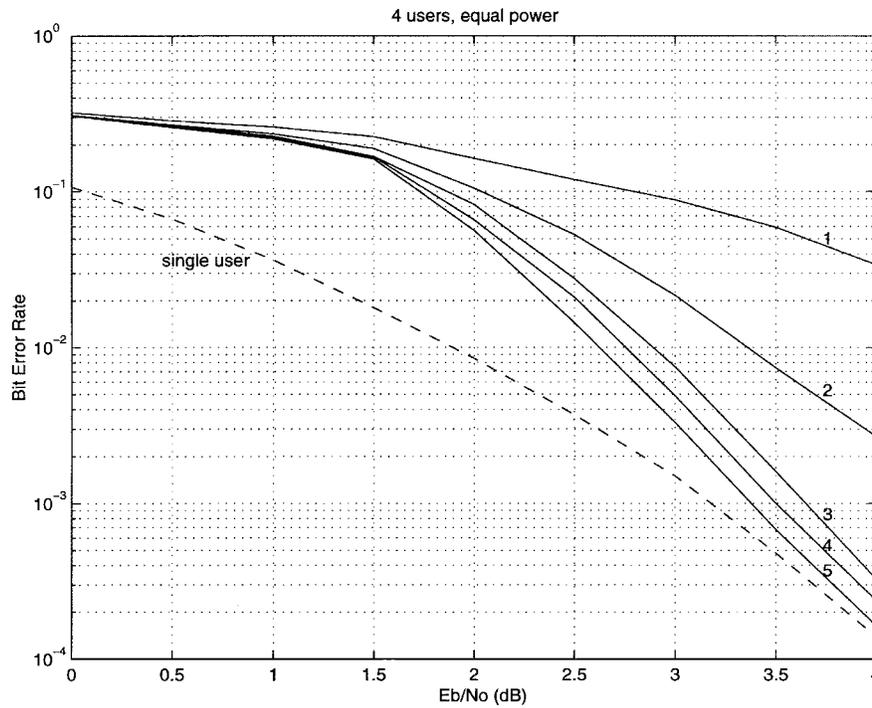


Fig. 2. Performance of the Turbo multiuser receiver that employs the exact SISO multiuser detector.  $K = 4$ ,  $\rho_{ij} = 0.7$ . All users have equal power.

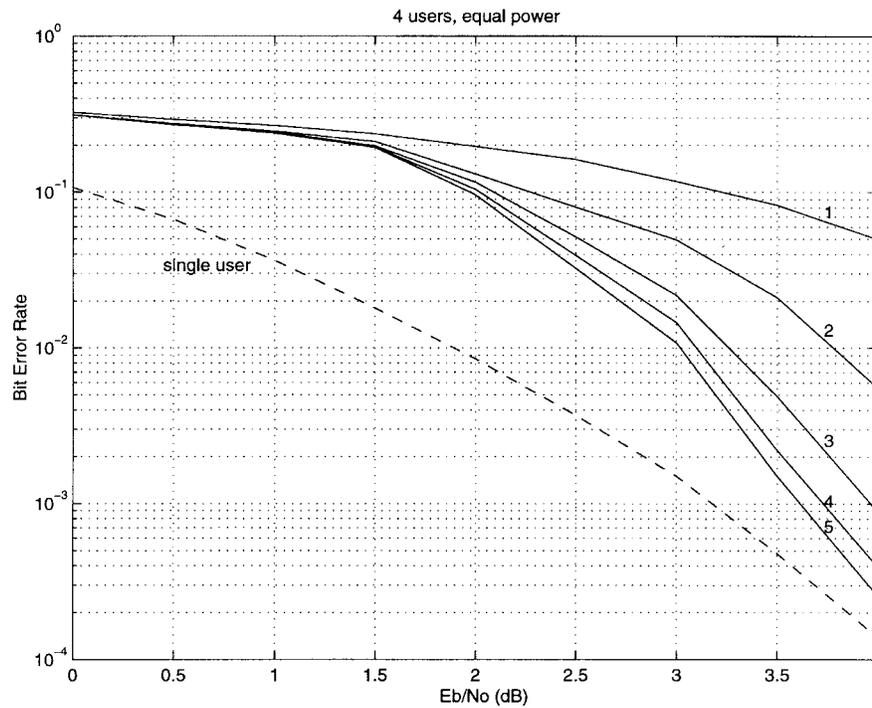


Fig. 3. Performance of the Turbo multiuser receiver that employs the approximate SISO multiuser detector.  $K = 4$ ,  $\rho_{ij} = 0.7$ . All users have equal power.

C. Simulation Results

In this section, we present some simulation examples to illustrate the performance of the Turbo multiuser receiver in synchronous CDMA systems. Of particular interest is the receiver that employs the low-complexity approximate SISO multiuser detector. All users employ the same rate 1/2 constraint length  $\nu = 5$  convolutional code (with generators 23, 35 in octal

notation). Each user uses a different interleaver generated randomly. The same set of interleavers is used for all simulations. The block size of the information bits for each user is 128.

First we consider a four-user system with equal cross-correlation  $\rho_{ij} = 0.7$ , for  $1 \leq i, j \leq 4$ . All the users have the same power. In Fig. 2 the performance of the Turbo receiver that employs the exact SISO multiuser detector (28) is shown for the first 5 iterations. In Fig. 3, the performance

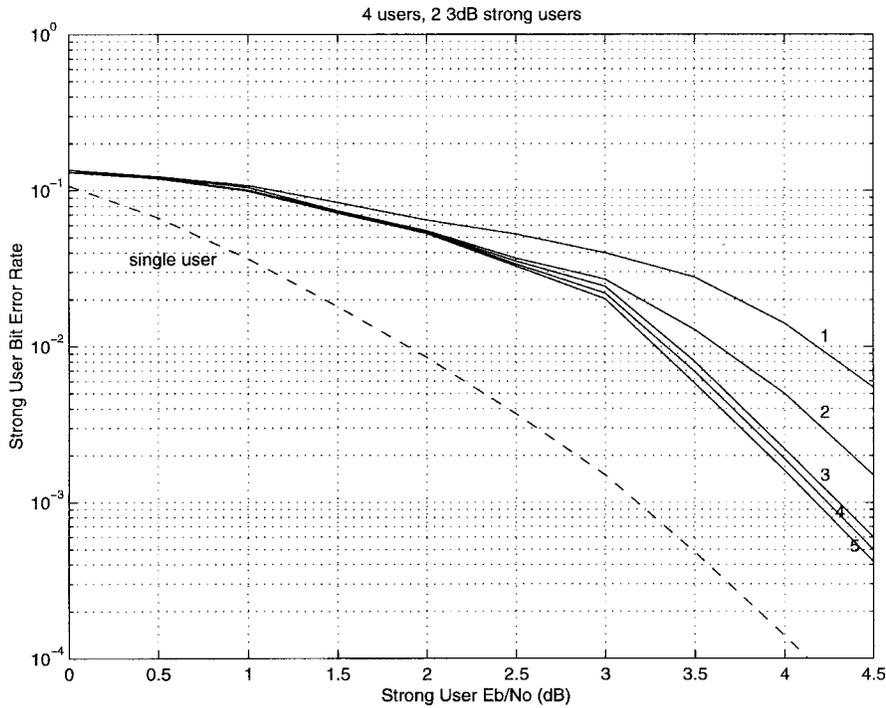


Fig. 4. Strong user performance under the Turbo multiuser receiver that employs the approximate SISO multiuser detector.  $K = 4$ ,  $\rho_{ij} = 0.7$ . Two users are 3 dB stronger than the other two.

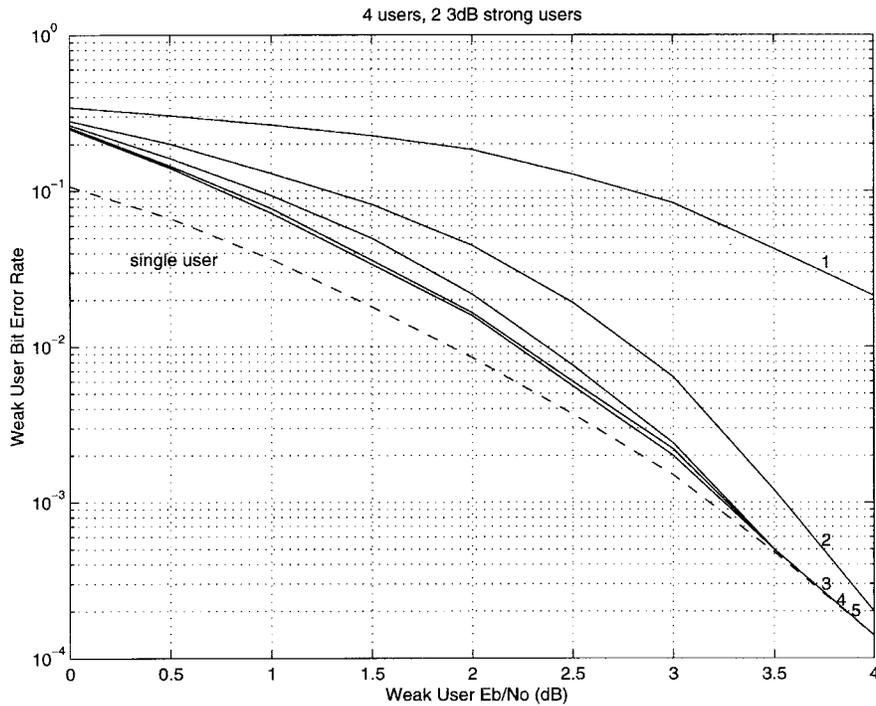


Fig. 5. Weak user performance under the Turbo multiuser receiver that employs the approximate SISO multiuser detector.  $K = 4$ ,  $\rho_{ij} = 0.7$ . Two users are 3 dB stronger than the other two.

of the Turbo receiver that employs the approximate SISO multiuser detector is shown for the same channel. In each of these figures, the single-user performance ( $\rho_{ij} = 0$ ) is also shown. It is seen that the performance of both receivers converges toward the single-user performance at high SNR. Moreover, the performance loss due to using the approximate

SISO multiuser detector is small. Next we consider a near-far situation, where there are two equal-power strong users and two equal-power weak users. The strong users' powers are 3 dB above the weak users'. The user cross correlations remain the same. Figs. 4 and 5 show, respectively, the performance of strong and weak users under the Turbo receiver that employs

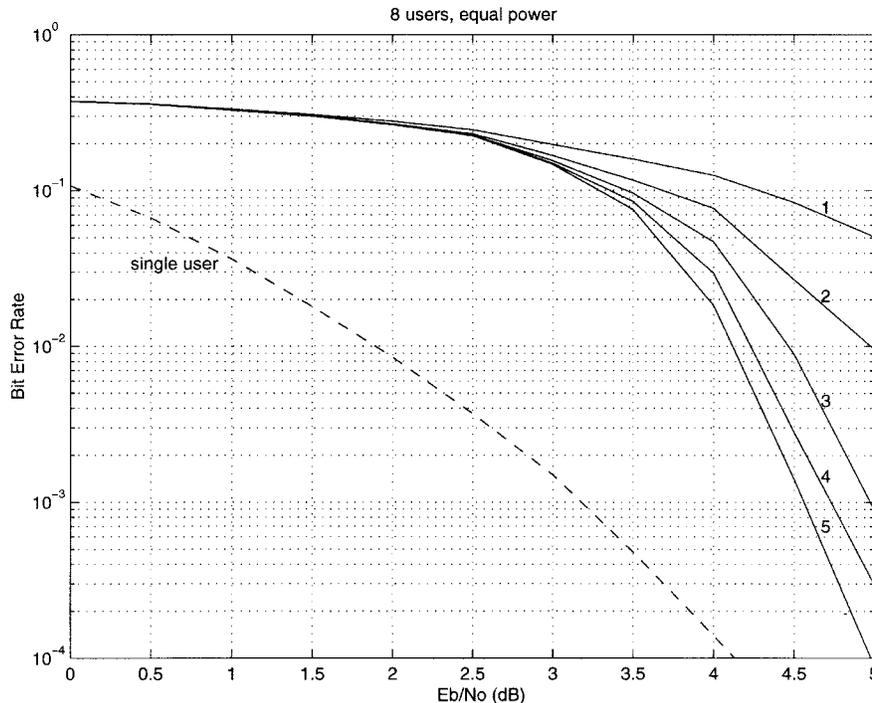


Fig. 6. Performance of the Turbo multiuser receiver that employs the approximate SISO multiuser detector.  $K = 8$ ,  $\rho_{ij} = 0.7$ . All users have equal power.

the approximate SISO multiuser detectors. It is seen that in such a near-far situation, the weak users actually benefit from the strong interference whereas the strong users suffer performance loss from the weak interference, a phenomenon previously also observed in the optimal multiuser detector [18] and the multistage multiuser detector [17]. Note that with a computational complexity  $O(2^K)$ , the exact SISO multiuser detector (28) is not feasible for practical implementation in channels with medium to large number of users  $K$ ; whereas the approximate SISO multiuser detector has a reasonable complexity that can be easily implemented even for very large  $K$ . Fig. 6 illustrates the performance of the Turbo receiver that employs the approximate SISO multiuser detector in a eight-user system. The user cross correlations are still  $\rho_{ij} = 0.7$ . All users have the same power. Note that the performance of such receiver after the first iteration corresponds to the performance of a “traditional” noniterative receiver structure consisting of a linear MMSE multiuser detector followed by  $K$  parallel (soft) channel decoders. It is seen from these figures that at reasonably high SNR, the proposed low-complexity iterative receiver offers significant performance gain over the traditional noniterative receiver.

## V. SISO MULTIUSER DETECTOR FOR MULTIPATH CHANNEL

### A. Discrete-Time Signal Model

In this section, we consider the SISO multiuser detectors for multipath CDMA channel. First we derive a discrete-time signal model for the multipath CDMA channel, which is instrumental in developing the SISO multiuser detection algorithms.

In the direct-sequence spread-spectrum multiple-access format, the user signaling waveforms are of the form

$$s_k(t) = \sum_{j=0}^{N-1} c_k(j)\psi(t - jT_c), \quad 0 \leq t \leq T \quad (55)$$

where  $N$  is the processing gain;  $\{c_k(j)\}_{j=0}^{N-1}$  is a signature sequence of  $\pm 1$ 's assigned to the  $k$ th user, and  $\psi$  is a normalized chip waveform of duration  $T_c = T/N$ . The received signal due to the  $k$ th user in (3) can then be written as

$$y_k(t) = A_k \sum_{i=0}^{M-1} b_k(i) \sum_{j=0}^{N-1} c_k(j)\tilde{g}_k(t - iT - jT_c) \quad (56)$$

where

$$\tilde{g}_k(t) \triangleq \psi(t) \star g_k(t) = \sum_{l=1}^{L_k} g_{kl}\psi(t - \tau_{kl}). \quad (57)$$

Define the discrete-time channel response for the  $k$ th user's signal as

$$\begin{aligned} f_k(m) &\triangleq \int_0^{T_c} \tilde{g}_k(t + mT_c)\psi(t) dt \\ &= \sum_{l=1}^{L_k} \int_0^{T_c} g_{kl}\psi(t - \tau_{kl} + mT_c)\psi(t) dt. \end{aligned} \quad (58)$$

Without loss of generality, assume that  $\tau_{k1} < \tau_{k2} < \dots < \tau_{kL_k}$ . Since the chip waveform  $\psi(t)$  is nonzero only on  $[0, T_c]$ ,  $f_k(m)$  is nonzero only for  $0 \leq m \leq \lceil \tau_{kL_k}/T_c \rceil$ . Furthermore, if the chip waveform  $\psi(t)$  is a normalized rectangle pulse

with duration  $T_c$ , then we have

$$f_k(m) = \sum_{l=1}^{L_k} g_{kl} \left[ \frac{(m+1)T_c - \tau_{kl}}{T_c} 1_{\{(m+1)T_c \leq \tau_{kl} < (m+2)T_c\}} \right. \\ \left. + \frac{\tau_{kl} - (m-1)T_c}{T_c} 1_{\{(m-1)T_c \leq \tau_{kl} < mT_c\}} \right]$$

where the indicator function  $1_{\{\Omega\}}$  is equal to one if event  $\Omega$  is true and zero, otherwise.

Let  $\{h_k(m)\}$  be the discrete-time composite signal waveform of the  $k$ th user, resulting from the convolution of the original spreading sequence  $\{c_k(m)\}$  with the total channel response  $\{f_k(m)\}$ , i.e.,

$$\{h_k(m)\} \triangleq A_k \{c_k(m)\} \star \{f_k(m)\}. \quad (59)$$

The length of the sequence  $\{h_k(m)\}$  is  $(N + \lceil \tau_{kL_k}/T_c \rceil)$ .

Denote  $\iota_k \triangleq 1 + \lceil \lceil \tau_{kL_k}/T_c \rceil / N \rceil$ , and  $\iota \triangleq \max_{1 \leq k \leq K} \{\iota_k\}$ . For  $0 \leq i < \iota$ , define an  $N \times K$  matrix

$$\underline{H}(i) \triangleq \begin{bmatrix} h_1(iN) & \cdots & h_K(iN) \\ \vdots & \vdots & \vdots \\ h_1(iN + N - 1) & \cdots & h_K(iN + N - 1) \end{bmatrix}_{N \times K}.$$

In order to convert the continuous-time received signal  $r(t)$  into a discrete-time signal, at the receiver,  $r(t)$  is first filtered by a chip-matched filter and then sampled at the chip rate. The resulting signal sample at the  $n$ th chip interval of the  $l$ th symbol interval is given by

$$r_n(i) \triangleq \int_{iT+nT_c}^{iT+(n+1)T_c} r(t) \psi(t - iT - nT_c) dt, \quad 0 \leq n < N. \quad (60)$$

Denote  $\underline{r}(i) \triangleq [r_0(i) \cdots r_{N-1}(i)]^T$ , and  $\underline{b}(i) \triangleq [b_1(i) \cdots b_K(i)]^T$ . Then we can write [20]

$$\underline{r}(i) = \underline{H}(i) \star \underline{b}(i) + \sigma \underline{n}(i) = \sum_{j=0}^{\iota-1} \underline{H}(j) \underline{b}(i-j) + \sigma \underline{n}(i). \quad (61)$$

where  $\underline{n}(i)$  is a complex Gaussian noise vector of dimension  $N$ ,  $\underline{n}(i) \sim \mathcal{N}_c(\underline{0}, \underline{I})$ .

By stacking  $\iota$  successive samples of the received data vector, we further define the following quantities:

$$\mathbf{r}(i) \triangleq \begin{bmatrix} \underline{r}(i) \\ \vdots \\ \underline{r}(i + \iota - 1) \end{bmatrix}_{N\iota \times 1}$$

$$\mathbf{b}(i) \triangleq \begin{bmatrix} \underline{b}(i - \iota + 1) \\ \vdots \\ \underline{b}(i + \iota - 1) \end{bmatrix}_{K(2\iota-1) \times 1}$$

$$\mathbf{n}(i) \triangleq \begin{bmatrix} \underline{n}(i) \\ \vdots \\ \underline{n}(i + \iota - 1) \end{bmatrix}_{N\iota \times 1}$$

$$\mathbf{H} \triangleq \begin{bmatrix} \underline{H}(\iota-1) & \cdots & \underline{H}(0) & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \underline{H}(\iota-1) & \cdots & \underline{H}(0) \end{bmatrix}_{N\iota \times K(2\iota-1)}$$

Then (61) can be written in matrix form as

$$\mathbf{r}(i) = \mathbf{H} \mathbf{b}(i) + \sigma \mathbf{n}(i). \quad (62)$$

where  $\mathbf{n}(i)$  is a complex Gaussian noise vector of dimension  $N\iota$ ,  $\mathbf{n}(i) \sim \mathcal{N}_c(\underline{0}, \mathbf{I})$ . Next we consider the exact as well as an approximate sliding-window SISO multiuser detector based on the discrete-time signal model (62).

### B. Sliding-Window SISO Multiuser Detector

A sliding-window multiuser detector makes decision on the  $i$ th multiuser code bit vector  $\underline{b}(i)$ , based on the signal  $\mathbf{r}(i)$ . Denote  $\mathcal{B}_k^+(i) \triangleq \{\mathbf{b}(i) \in \{-1, +1\}^{K(2\iota-1)} : b_k(i) = +1\}$ . Similarly define  $\mathcal{B}_k^-(i)$ . The extrinsic information  $\lambda_1[b_k(i)]$  delivered by the exact sliding-window SISO multiuser detector is given by (63), shown at the bottom of the page, where we denote

$$\mathbf{y}(i) \triangleq \Re\{\mathbf{H}^H \mathbf{r}\} \\ = [\underline{y}(i - \iota + 1)]^T \cdots \underline{y}(i)^T \cdots \underline{y}(i + \iota - 1)]^T \quad (64)$$

$$\underline{y}(l) = [y_1(l) \cdots y_K(l)]^T, \quad i - \iota + 1 \leq l \leq i + \iota - 1. \quad (65)$$

The number of terms to be summed in the numerator (denominator) of (63) is  $2^{K(2\iota-1)-1}$ . Similar to what is discussed

$$\lambda_1[b_k(i)] = \log \frac{p[\mathbf{r}(i) | b_k(i) = +1]}{p[\mathbf{r}(i) | b_k(i) = -1]} \\ = \log \frac{\sum_{\mathbf{b}(i) \in \mathcal{B}_k^+} \exp[-\|\mathbf{r}(i) - \mathbf{H} \mathbf{b}(i)\|^2 / \sigma^2] \prod_{(j,l) \neq (k,i)} P[b_j(l)]}{\sum_{\mathbf{b}(i) \in \mathcal{B}_k^-} \exp[-\|\mathbf{r}(i) - \mathbf{H} \mathbf{b}(i)\|^2 / \sigma^2] \prod_{(j,l) \neq (k,i)} P[b_j(l)]} \\ = \log \frac{\sum_{\mathbf{b}(i) \in \mathcal{B}_k^+} \exp[-\|\mathbf{H} \mathbf{b}\|^2 / \sigma^2] \exp[2\Re\{\mathbf{b}(i)^T \mathbf{y}(i)\} / \sigma^2] \prod_{(j,l) \neq (k,i)} P[b_j(l)]}{\sum_{\mathbf{b}(i) \in \mathcal{B}_k^-} \exp[-\|\mathbf{H} \mathbf{b}\|^2 / \sigma^2] \exp[2\Re\{\mathbf{b}(i)^T \mathbf{y}(i)\} / \sigma^2] \prod_{(j,l) \neq (k,i)} P[b_j(l)]} \\ = \log \frac{\sum_{\mathbf{b}(i) \in \mathcal{B}_k^+} \exp[-\|\mathbf{H} \mathbf{b}\|^2 / \sigma^2] \prod_{(j,l) \neq (k,i)} [1 + b_j(l) \tanh(2y_j(l) / \sigma^2)] [1 + b_j(l) \tanh(\frac{1}{2} \lambda_2^2[b_j(l)])]}{\sum_{\mathbf{b}(i) \in \mathcal{B}_k^-} \exp[-\|\mathbf{H} \mathbf{b}\|^2 / \sigma^2] \prod_{(j,l) \neq (k,i)} [1 + b_j(l) \tanh(2y_j(l) / \sigma^2)] [1 + b_j(l) \tanh(\frac{1}{2} \lambda_2^2[b_j(l)])]} + 4y_k(i) / \sigma^2 \quad (63)$$

for the synchronous CDMA system, we next consider a low-complexity approximate sliding-window SISO multiuser detector, based on soft interference cancellation and instantaneous linear MMSE filtering.

### C. Low-Complexity Sliding-Window SISO Multiuser Detector

Based on the *a priori* LLR of the code bits of all users,  $\lambda_2^p[b_j(i)]$ ,  $1 \leq j \leq K$ ,  $0 \leq i < M - 1$ , provided by the SISO channel decoder from the previous stage, we first form soft estimates of the user code bits  $\tilde{b}_j(i) = \tanh(\frac{1}{2}\lambda_2^p[b_j(i)])$ . Denote

$$\tilde{\mathbf{b}}(i) = [\tilde{b}_1(i) \quad \dots \quad \tilde{b}_K(i)]^T \quad (66)$$

$$\tilde{\mathbf{b}}(i) = [\tilde{b}(i - \ell + 1)^T \quad \dots \quad \tilde{b}(i)^T \quad \dots \quad \tilde{b}(i + \ell - 1)^T]^T \quad (67)$$

and

$$\tilde{\mathbf{b}}_k(i) \triangleq \tilde{\mathbf{b}}(i) - \tilde{b}_k(i)\mathbf{e}_k \quad (68)$$

where  $\mathbf{e}_k$  denotes a  $[K(2\ell - 1)]$ -vector of all zeros, except for the  $[(\ell - 1)K + k]$ th element, which is 1.

At symbol time  $i$ , for each user  $k$ , a soft interference cancellation is performed on the received discrete-time signal  $\mathbf{r}(i)$  in (62), to obtain

$$\mathbf{r}_k(i) \triangleq \mathbf{r}(i) - \mathbf{H}\tilde{\mathbf{b}}_k(i) = \mathbf{H}[\mathbf{b}(i) - \tilde{\mathbf{b}}_k(i)] + \sigma\mathbf{n}(i), \quad k = 1, \dots, K. \quad (69)$$

An instantaneous linear MMSE filter is then applied to  $\mathbf{r}_k(i)$ , to obtain

$$z_k(i) = \mathbf{w}_k(i)^H \mathbf{r}_k(i) \quad (70)$$

where the filter  $\mathbf{w}_k(i) \in \mathcal{C}^{N_\ell}$  is chosen to minimize the mean-square error between the code bit  $b_k(i)$  and the filter output  $z_k(i)$ , i.e.,

$$\begin{aligned} \mathbf{w}_k(i) &= \arg \min_{\mathbf{w} \in \mathcal{C}^{N_\ell}} E\{\|b_k(i) - \mathbf{w}^H \mathbf{r}_k(i)\|^2\} \\ &= \arg \min_{\mathbf{w} \in \mathcal{C}^{N_\ell}} \mathbf{w}^H E\{\mathbf{r}_k(i)\mathbf{r}_k(i)^H\} \mathbf{w} \\ &\quad - \mathbf{w}^H E\{b_k(i)\mathbf{r}_k(i)\} - E\{b_k(i)\mathbf{r}_k(i)\}^H \mathbf{w} \end{aligned} \quad (71)$$

where

$$E\{\mathbf{r}_k(i)\mathbf{r}_k(i)^H\} = \mathbf{H}\Delta_k(i)\mathbf{H}^H + \sigma^2\mathbf{I} \quad (72)$$

$$E\{[b_k(i)\mathbf{r}_k(i)]\} = \mathbf{H}\mathbf{e}_k \triangleq \mathbf{h}_k \quad (73)$$

and

$$\begin{aligned} \Delta_k(i) &\triangleq \text{cov}\{\mathbf{b}(i) - \tilde{\mathbf{b}}_k(i)\} \\ &= \text{diag}[\Delta_k(i - \ell + 1), \dots, \Delta_k(i), \dots, \Delta_k(i + \ell - 1)] \end{aligned}$$

$$\begin{aligned} \Delta_k(i - \ell) &\triangleq \text{diag}[1 - \tilde{b}_1(i - \ell)^2, \dots, \\ &\quad 1 - \tilde{b}_K(i - \ell)^2], \quad l = \pm 1, \dots, \pm(\ell - 1) \end{aligned}$$

$$\begin{aligned} \Delta_k(i) &\triangleq \text{diag}[1 - \tilde{b}_1(i)^2, \dots, 1 - \tilde{b}_{k-1}(i)^2, 1, \\ &\quad 1 - \tilde{b}_{k+1}(i)^2, 1 - \tilde{b}_K(i)^2]. \end{aligned}$$

The solution to (71) is given by

$$\mathbf{w}_k(i) = [\mathbf{H}\Delta_k(i)\mathbf{H}^H + \sigma^2\mathbf{I}]^{-1} \mathbf{h}_k. \quad (74)$$

As before, in order to form the LLR of the code bit  $b_k(i)$ , we approximate the soft instantaneous MMSE filter output  $z_k(i)$  in (70) as Gaussian distributed, i.e.,  $z_k(i) \sim \mathcal{N}_c(\mu_k(i)b_k(i), \nu_k^2(i))$ . Conditioned on the code bit  $b_k(i)$ , the mean and variance of  $z_k(i)$  are given respectively by

$$\begin{aligned} \mu_k(i) &\triangleq E\{z_k(i)b_k(i)\} \\ &= \mathbf{e}_k^H \mathbf{H}^H [\mathbf{H}\Delta_k(i)\mathbf{H}^H + \sigma^2\mathbf{I}]^{-1} \mathbf{H}E\{\mathbf{b}(i) - \tilde{\mathbf{b}}_k(i)\} \\ &= \mathbf{h}_k^H [\mathbf{H}\Delta_k(i)\mathbf{H}^H + \sigma^2\mathbf{I}]^{-1} \mathbf{h}_k \end{aligned} \quad (75)$$

$$\begin{aligned} \nu_k^2(i) &\triangleq \text{var}\{z_k(i)\} = E\{\|z_k(i)\|^2\} - \mu_k(i)^2 \\ &= \mathbf{w}_k^H E\{\mathbf{r}_k(i)\mathbf{r}_k(i)^H\} \mathbf{w}_k - \mu_k^2 \\ &= \mathbf{h}_k^H [\mathbf{H}\Delta_k(i)\mathbf{H}^H + \sigma^2\mathbf{I}]^{-1} \mathbf{h}_k - \mu_k(i)^2 \\ &= \mu_k(i) - \mu_k(i)^2. \end{aligned} \quad (76)$$

Therefore the extrinsic information  $\lambda_1[b_k(i)]$  delivered by the soft instantaneous MMSE filter is given by

$$\begin{aligned} \lambda_1[b_k(i)] &= -\frac{|z_k - \mu_k|^2}{\nu_k^2} + \frac{|z_k - \mu_k|^2}{\nu_k^2} \\ &= \frac{4\Re\{\mu_k(i)z_k(i)\}}{\nu_k^2(i)} = \frac{4\Re\{z_k(i)\}}{1 - \mu_k(i)}. \end{aligned} \quad (77)$$

The SINR at the soft instantaneous MMSE filter output is given by

$$\text{SINR}[z_k(i)] \triangleq \frac{E^2\{\Re\{z_k(i)\}\}}{\text{var}\{\Re\{z_k(i)\}\}} = \frac{\mu_k(i)^2}{\frac{1}{2}\nu_k^2(i)} = \frac{2}{1/\mu_k(i) - 1}. \quad (78)$$

It is easily seen that when no prior information on the users' code bits is available,  $\Delta_k(i) = \mathbf{I}$ ; when perfect information about the code bits is available,  $\Delta_k(i) = \mathbf{e}_k\mathbf{e}_k^T$ . As before, we denote  $\overline{\text{SINR}}[z_k(i)]$  as the output SINR when there is no prior information on the code bits of interfering users; and denote  $\underline{\text{SINR}}[z_k(i)]$  as the output SINR when there is perfect prior information on the code bits of interfering users. In particular, when perfect information on code bits is available, we have

$$\bar{\mu}_k(i) = \mathbf{h}_k^H (\mathbf{h}_k\mathbf{h}_k^H + \sigma^2\mathbf{I})^{-1} \mathbf{h}_k = \frac{\|\mathbf{h}_k\|^2}{\sigma^2 + \|\mathbf{h}_k\|^2} \quad (79)$$

$$\overline{\text{SINR}}[z_k(i)] = \frac{2}{1/\bar{\mu}_k(i) - 1} = \frac{2\|\mathbf{h}_k\|^2}{\sigma^2}. \quad (80)$$

As expected, in this case  $\overline{\text{SINR}}[z_k(i)]$  is simply the input SNR of the  $k$ th user. In general, the prior information on the multiuser code bits provided by the SISO channel decoders satisfy  $0 < |\lambda_2^p[b_j(i)]| < \infty$ . Then it is shown in Appendix B that

$$\overline{\text{SINR}}[z_k(i)] \geq \text{SINR}[z_k(i)] \geq \underline{\text{SINR}}[z_k(i)]. \quad (81)$$

The major computation involved in computing the soft MMSE filter output is the matrix inversion  $\Phi_k(i) \triangleq [\mathbf{H}\Delta_k(i)\mathbf{H}^H + \sigma^2\mathbf{I}]^{-1}$ . Since the matrix  $\mathbf{H}\Delta_k(i)\mathbf{H}^H$  is the sum of  $[K(2\ell - 1)]$  vector products, by applying the matrix inversion lemma, we can compute this matrix inversion recursively. A recursive procedure for computing this matrix

TABLE I

RECURSIVE PROCEDURE FOR COMPUTING  $\Phi_k(i)$ . [ $\mathbf{H}(:, l)$  DENOTES THE  $l$ -TH COLUMN OF  $\mathbf{H}$ ]. THE DOMINANT COMPUTATION IN THE ABOVE RECURSION INCLUDES THE  $(2l - 1)K$  MATRIX-VECTOR PRODUCTS AND THE  $(2l - 1)K$  VECTOR OUTER PRODUCTS IN (\*), AS WELL AS THE  $K$  MATRIX-VECTOR PRODUCTS AND  $K$  VECTOR OUTER PRODUCTS IN (\*\*). THEREFORE, IN COMPUTING THE SOFT OUTPUT OF THE APPROXIMATE SLIDING-WINDOW SISO MULTIUSER DETECTOR, THE DOMINANT COMPUTATION PER USER PER SYMBOL INVOLVES  $(2l)$  MATRIX-VECTOR PRODUCTS (MATRIX DIMENSION:  $Nl \times Nl$ ), AND  $(2l)$  VECTOR OUTER PRODUCTS (VECTOR DIMENSION:  $Nl$ )

$\Psi^{(0)}(i) \triangleq \mathbf{I}/\sigma^2$
for $l = 1, 2, \dots, (2l - 1)K$
$\kappa(l) \triangleq [(l - 1) \bmod K] + 1$
$\delta(l) \triangleq [(l - 1)/K] - (l - 1)$
$\alpha(l) \triangleq [1 - \tilde{b}_{\kappa(l)}(i - \delta(l))]^{-1} + \mathbf{H}(:, l)^H \Psi^{(l-1)}(i) \mathbf{H}(:, l)$
$\Psi^{(l)}(i) \triangleq (\sigma^2 \mathbf{I} + \sum_{j=1}^l [1 - \tilde{b}_{\kappa(l)}(i - \delta(l))] \mathbf{H}(:, l) \mathbf{H}(:, l)^H)^{-1}$
$= \Psi^{(l-1)}(i) - \frac{1}{\alpha(l)} [\Psi^{(l-1)}(i) \mathbf{H}(:, l)] [\Psi^{(l-1)}(i) \mathbf{H}(:, l)]^H \quad (*)$
end
$\Psi \triangleq \Psi^{(2l-1)K}(i)$
for $k = 1, 2, \dots, K$
$j(k) \triangleq (l - 1)K + k$
$\beta(k) \triangleq \tilde{b}_k(i)^2 + \mathbf{H}(:, j(k))^H \Psi \mathbf{H}(:, j(k))$
$\Phi_k(i) = [\Psi^{-1} + \tilde{b}_k(i)^2 \mathbf{H}(:, j(k)) \mathbf{H}(:, j(k))^H]^{-1}$
$= \Psi - \frac{1}{\beta(k)} [\Psi \mathbf{H}(:, j(k))] [\Psi \mathbf{H}(:, j(k))]^H \quad (**)$
end

TABLE II  
SIMULATED MULTIPATH CDMA SYSTEM

User #	Signature $\{c_k(j)\}$	Multipath delay ( $T_c$ )			Multipath gain		
		$\tau_{k1}$	$\tau_{k2}$	$\tau_{k3}$	$g_{k1}$	$g_{k2}$	$g_{k3}$
1	0101110	1.618	2.840	4.248	$-0.291 + j 0.501$	$0.361 - j 0.506$	$0.528 - j 0.008$
2	1001110	3.402	5.335	6.239	$-0.578 - j 0.057$	$-0.523 - j 0.582$	$0.207 + j 0.093$
3	1000010	0.130	3.195	5.750	$-0.178 - j 0.469$	$0.307 + j 0.629$	$0.362 - j 0.358$
4	1100000	2.470	3.113	4.308	$0.712 + j 0.529$	$1.290 + j 0.219$	$0.669 - j 0.922$

inversion is outlined in Table I. By employing this recursion, in computing the soft output of the approximate sliding-window SISO multiuser detector, the dominant computation per user per symbol involves  $(2l)$  matrix-vector products (matrix dimension:  $Nl \times Nl$ ), and  $(2l)$  vector outer products (vector dimension:  $Nl$ ). The total computational complexity of the Turbo multiuser detector is then  $O(N^2 l^2 + 2^\nu)$ , where  $\nu$  is the code constraint length.

#### D. Simulation Results

In this section we illustrate the performance of the sliding-window Turbo multiuser receiver that employs the approximate SISO multiuser detector in a multipath CDMA channel. We consider an asynchronous CDMA system with four users ( $K = 4$ ). The user spreading sequences are derived from Gold sequences of length seven ( $N = 7$ ), as used in [17]. The multipath channel model is given by (2). The number of paths for each user is three ( $L_k = 3$ ). In Table II we list the signature sequence  $\{c_k(j)\}$ , path delays  $\{\tau_{kl}\}$  and complex path gains

$\{g_{kl}\}$  for each user  $k$ . The multipath delays are in terms of number of chip intervals ( $T_c$ ). The complex path gains for each user are normalized such that the composite signature sequence  $\mathbf{h}_k$  satisfies  $\|\mathbf{h}_k\|/A_k = 1$  [cf. (59)]. As before all users employ the same rate  $\frac{1}{2}$  constraint length 5 convolutional code. Each user uses a different random interleaver. The same set of interleavers is used for all simulations. The block size of the information bits for each user is 128. In the simulation, the four user signals have equal power. The bit error rate curves of users 1 and 2 are shown in Figs. 7 and 8. It is seen that significant performance gain is achieved by the proposed iterative receiver structure compared with the noniterative receiver structure (i.e., linear MMSE demodulator followed by soft channel decoder). Moreover, at high SNR, the detrimental effects of the MAI and ISI in the channel can almost be completely eliminated and single-user performance can be approached. Note that similar performance gain by a Turbo equalizer in a single-user ISI channel has been previously reported in [4].

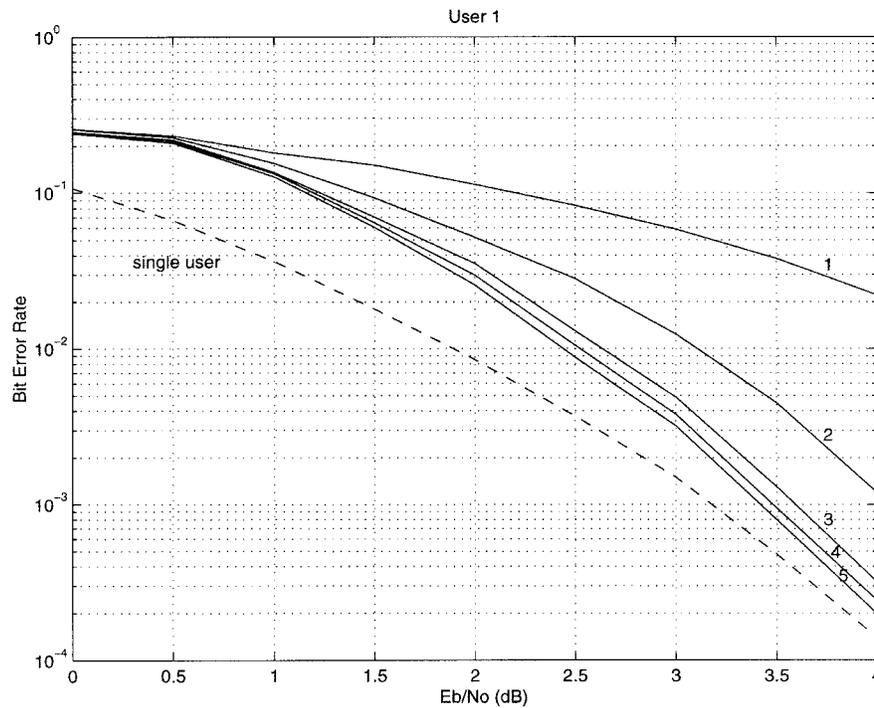


Fig. 7. User 1 performance of the sliding-window Turbo multiuser receiver that employs the approximate SISO multiuser detector in a four-user multipath channel. All users have equal power.  $N = 7$ .

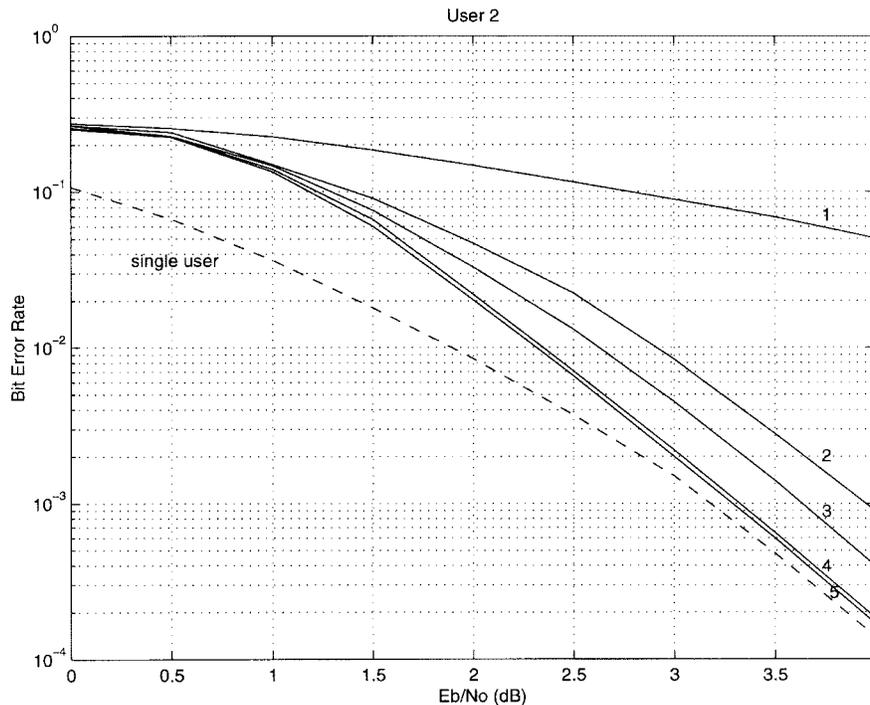


Fig. 8. User 2 performance of the sliding-window Turbo multiuser receiver that employs the approximate SISO multiuser detector in a four-user multipath channel. All users have equal power.  $N = 7$ .

## VI. CONCLUSION

There is currently a significant interest in the design of wide-band CDMA networks which would give users access to data rates on the order of 1 to 10's of Mb/s [5], [14], or even higher ATM (asynchronous transfer mode)-compatible rates for wireless multimedia applications [12]. In a high-

rate CDMA system, the MAI and the ISI constitute the major impediment to the overall system performance. In this paper, we have developed a low-complexity iterative receiver structure for decoding multiuser information data in a convolutionally coded asynchronous multipath DS-CDMA system. At each iteration, extrinsic information is extracted from detection and decoding stages and is then used as a priori

information in the next iteration, just as in Turbo decoding. A low-complexity SISO multiuser detector is developed based on a novel nonlinear interference suppression technique, which makes use of both soft-interference cancellation and instantaneous linear MMSE filtering. Simulation results demonstrate that the proposed low-complexity Turbo multiuser receiver offers performance approaching that of the single-user channel at high SNR.

#### APPENDIX A PROOF OF (46)

First we have the following.

*Fact:* Let  $\underline{X}$  be a  $K \times K$  positive definite matrix. Denote  $\underline{X}_{\setminus k}$  be the  $(K-1) \times (K-1)$  matrix obtained from  $\underline{X}$  by deleting the  $k$ th row and  $k$ th column. Denote also  $\underline{x}_k$  the  $k$ th column of  $\underline{X}$  with the  $k$ th entry  $x_{kk}$  removed. Then we have

$$[\underline{X}^{-1}]_{kk} = \frac{1}{1 - \underline{x}_k^T \underline{X}_{\setminus k}^{-1} \underline{x}_k}. \quad (82)$$

*Proof:* Since  $\underline{X}_{\setminus k}$  is a principle submatrix of  $\underline{X}$ , and  $\underline{X}$  is positive definite,  $\underline{X}_{\setminus k}$  is also positive definite. Hence  $\underline{X}_{\setminus k}^{-1}$  exists.

Denote the above-mentioned partitioning of the symmetric matrix  $\underline{X}$  with respect to the  $k$ th column and row by  $\underline{X} = \begin{pmatrix} \underline{X}_{\setminus k} & \underline{x}_k \\ \underline{x}_k^T & x_{kk} \end{pmatrix}$ . In the same way we partition its inverse  $\underline{Y} \triangleq \underline{X}^{-1} = \begin{pmatrix} \underline{Y}_{\setminus k} & \underline{y}_k \\ \underline{y}_k^T & y_{kk} \end{pmatrix}$ . Now from the fact that  $\underline{X}\underline{Y} = \underline{I}$ , it follows that

$$\underline{X}_{\setminus k} \underline{y}_k + y_{kk} \underline{x}_k = \underline{0} \quad (83)$$

$$\underline{y}_k^T \underline{x}_k + y_{kk} = 1. \quad (84)$$

Solving  $y_{kk}$  from (83) and (84), we obtain (82).  $\square$

Using (48) and (49), by definition we have

$$\text{SINR}[z_k(i)] = \frac{\mu_k(i)^2}{\nu_k^2(i)} = \frac{1}{1/\mu_k(i) - 1}. \quad (85)$$

From (48) and (85) it is immediate that (46) is equivalent to

$$\begin{aligned} [(A_k^2 \underline{e}_k \underline{e}_k^T + \sigma^2 \underline{R}^{-1})^{-1}]_{kk} &> [(\underline{V}_k(i) + \sigma^2 \underline{R}^{-1})^{-1}]_{kk} \\ &> [(\underline{A}^2 + \sigma^2 \underline{R}^{-1})^{-1}]_{kk}. \end{aligned} \quad (86)$$

Partition the three matrices above with respect to the  $k$ th column and  $k$ th row to get  $(A_k^2 \underline{e}_k \underline{e}_k^T + \sigma^2 \underline{R}^{-1}) = (\underline{Q}_k, \underline{q}_k, \alpha)$ ,  $(\underline{V}_k(i) + \sigma^2 \underline{R}^{-1}) = (\underline{P}_k, \underline{p}_k, \beta)$ , and  $(\underline{A}^2 + \sigma^2 \underline{R}^{-1}) = (\underline{Q}_k, \underline{q}_k, \gamma)$ . By (82), (86) is then equivalent to

$$\underline{q}_k^T \underline{Q}_k^{-1} \underline{q}_k > \underline{p}_k^T \underline{P}_k^{-1} \underline{p}_k > \underline{q}_k^T \underline{Q}_k^{-1} \underline{q}_k. \quad (87)$$

Since  $\underline{A}^2 = \sum_{j=1}^K A_j^2 \underline{e}_j \underline{e}_j^T$ ,  $\underline{V}_k(i) = A_k^2 \underline{e}_k \underline{e}_k^T + \sum_{j \neq k} A_j^2 [1 - \tilde{b}_j(i)^2] \underline{e}_j \underline{e}_j^T$ , we have  $\underline{q}_k = \underline{p}_k = \underline{q}_k$ . Therefore in order to show (87) it suffices to show  $\underline{Q}_k^{-1} \succ \underline{P}_k^{-1} \succ \underline{Q}_k^{-1}$ , which is in turn equivalent to  $\underline{Q}_k \succ \underline{P}_k \succ \underline{Q}_k$  [10], where  $\underline{X} \succ \underline{Y}$  means that the matrix  $\underline{X} - \underline{Y}$  is positive definite. Since by assumption,  $0 < |\lambda_2^p[b_j(i)]| < \infty$ ,  $j = 1, \dots, K$ , therefore

$0 < \tilde{b}_j(i) < 1$ ,  $j = 1, \dots, K$ . It is easy to check that

$$\begin{aligned} \underline{Q}_k - \underline{P}_k &= \text{diag}[A_1^2 \tilde{b}_1(i)^2, \dots, A_{k-1}^2 \tilde{b}_{k-1}(i)^2, \\ &\quad A_{k+1}^2 \tilde{b}_{k+1}(i)^2, \dots, A_K^2 \tilde{b}_K(i)^2] \succ \underline{0}, \\ \underline{P}_k - \underline{Q}_k &= \text{diag}[A_1^2 [1 - \tilde{b}_1(i)^2], \dots, A_{k-1}^2 [1 - \tilde{b}_{k-1}(i)^2], \\ &\quad A_{k+1}^2 [1 - \tilde{b}_{k+1}(i)^2], \dots, A_K^2 [1 - \tilde{b}_K(i)^2]] \\ &\succ \underline{0}. \end{aligned} \quad \square$$

#### APPENDIX B PROOF OF (81)

By (75) and (78), in order to show (81), it suffices to show that

$$\begin{aligned} [\mathbf{h}_k \mathbf{h}_k^H + \sigma^2 \mathbf{I}]^{-1} &\succ [\mathbf{H} \underline{\Delta}_k(i) \mathbf{H}^H + \sigma^2 \mathbf{I}]^{-1} \\ &\succ [\mathbf{H} \mathbf{H}^H + \sigma^2 \mathbf{I}]^{-1}. \end{aligned} \quad (88)$$

This is equivalent to [10]

$$\begin{aligned} \mathbf{H} \mathbf{H}^H + \sigma^2 \mathbf{I} \succ \mathbf{H} \underline{\Delta}_k(i) \mathbf{H}^H + \sigma^2 \mathbf{I} &\succ \mathbf{h}_k \mathbf{h}_k^H + \sigma^2 \mathbf{I} \Leftrightarrow \\ \mathbf{H} \mathbf{H}^H \succ \mathbf{H} \underline{\Delta}_k(i) \mathbf{H}^H &\succ \mathbf{h}_k \mathbf{h}_k^H, \end{aligned}$$

which is true by the definitions of  $\underline{\Delta}_k(i)$  and  $\mathbf{h}_k$ .  $\square$

#### REFERENCES

- [1] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. IT-20, pp. 284-287, Mar. 1974.
- [2] C. Berrou and A. Glavieux, "Near optimum error-correcting coding and decoding: Turbo codes," *IEEE Trans. Commun.*, vol. 44, Oct. 1996.
- [3] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correction coding and decoding: Turbo codes," in *Proc. 1993 Int. Conf. on Communications (ICC'93)*, 1993, pp. 1064-1070.
- [4] C. Douillard et al., "Iterative correction of intersymbol interference: Turbo equalization," *European Trans. Telecommun.*, vol. 6, no. 5, pp. 507-511, Sept.-Oct. 1995.
- [5] R. O. LaMaire et al., "Wireless LAN's and mobile networking: Standard and future directions," *IEEE Commun. Mag.*, vol. 35, no. 8, pp. 86-94, Aug. 1996.
- [6] T. R. Giallorenzi and S. G. Wilson, "Multiuser ML sequence estimator for convolutional coded asynchronous DS-CDMA systems," *IEEE Trans. Commun.*, vol. COM-44, pp. 997-1008, Aug. 1996.
- [7] T. R. Giallorenzi and S. G. Wilson, "Suboptimum multiuser receivers for convolutionally coded asynchronous DS-CDMA systems," *IEEE Trans. Commun.*, vol. COM-44, no. 9, pp. 1183-1196, Sept. 1996.
- [8] J. Hagenauer, "Forward error correcting for CDMA systems," in *Proc. IEEE Fourth Int. Symp. on Spread Spectrum Techniques and Applications (ISSSTA'96)*, Mainz, Germany, Sept. 1996, pp. 566-569.
- [9] J. Hagenauer, "The Turbo principle: Tutorial introduction and state of the art," in *Proc. International Symposium on Turbo Codes and Related Topics*, Brest, France, Sept. 1997, pp. 1-11.
- [10] R. A. Horn and C. R. Johnson. *Matrix Analysis*, Cambridge University Press, 1985.
- [11] M. Moher, "An iterative multiuser decoder for near-capacity communications," *IEEE Trans. Commun.*, vol. 46, pp. 870-880, July 1998.
- [12] N. Morinaga, M. Nakagawa, and R. Kohno, "New concepts and technologies for achieving highly reliable and high-capacity multimedia wireless communications systems," *IEEE Commun. Mag.*, vol. 36, pp. 34-40, Jan. 1997.
- [13] R. A. Müller and J. B. Huber, "Iterated soft decision interference cancellation for CDMA," in *Proc. 9th Tyrrhenian Int. Workshop on Digital Comm.*, Italy, 1997.
- [14] K. Pahlavan, T. J. Probert, and M. E. Chase, "Trends in local wireless networks," *IEEE Commun. Mag.*, vol. 34, no. 3, pp. 88-95, Mar. 1995.
- [15] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Trans. Inform. Theory*, vol. IT-43, pp. 858-871, May 1997.
- [16] M. C. Reed et al., "Iterative multiuser detection for CDMA with FEC: Near single user performance," *IEEE Trans. Commun.*, vol. 46, pp. 1693-1699, Dec. 1998.

- [17] F. Tarköy, "Iterative multiuser decoding for asynchronous users," in *Proc. 1997 IEEE Int. Symp. on Inform. Theory (ISIT'97)*, Ulm, Germany, June 1997, p. 30.
- [18] M. K. Varanasi and B. Aazhang, "Near-optimum detection in synchronous code-division multiple-access systems," *IEEE Trans. Commun.*, vol. 39, pp. 725–736, May 1991.
- [19] S. Verdú, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, vol. IT-32, pp. 85–96, Jan. 1986.
- [20] S. Verdú, *Multiuser Detection*, Cambridge, U.K.: Cambridge Univ. Press, 1998.
- [21] X. Wang and H. V. Poor, "Blind equalization and multiuser detection for CDMA communications in dispersive channels," *IEEE Trans. Commun.*, vol. 46, pp. 91–103, Jan. 1998.



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